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Existence and uniqueness of Cournot equilibrium for an oligopoly under linear and nonlinear demand: a channel model for one manufacturer-two retailers with substitutes

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Existence and uniqueness of Cournot equilibrium for an oligopoly under linear and nonlinear demand: A channel model for one manufacturer-two retailers with substitutes

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ABSTRACT

This paper considers the margin, price, and quantity decisions made by one manufacturer and two retailers in a two-echelon supply chain under Cournot competition. The manufacturer produces two types of substitutable products and sells one product to each retailer. The retailers carry one type of product, which is sold to the customer. The objective of this paper is to show existence and uniqueness of Cournot equilibrium for decisions made by one manufacturer and two retailers and to study the implications of these decisions through sensitivity analysis. This paper makes contributions to the literature in three ways. First, it extends on the literature by modeling an oligopoly of one manufacturer and two retailers selling differentiated products and acting in Cournot competition. Second, this paper demonstrates the sufficient conditions for existence and uniqueness of Cournot equilibrium in a two-echelon supply chain with linear and nonlinear demand. Third, this paper has three main findings: (1) the retailer with the lower price effect or lower price elasticity of demand has a higher margin, price, quantity demanded, and profit; (2) all players in the supply chain prefer to sell products with a high cross price effect or high cross price elasticity of demand; and (3) the manufacturer will always earn the most profit among the three members of the supply chain.

CHAPTER 1. INTRODUCTION

This paper examines the margin, price, and quantity decisions made by an oligopoly under Cournot competition. The oligopoly, which was also modeled by Yang & Zhou (2006), is a supply chain that consists of one manufacturer and two retailers and is represented in Figure 1.1. The manufacturer produces two types of products, and one type of product is sold to each retailer for the same wholesale price. The products sold to the customers are substitutable. For example, suppose a food manufacturer produces cheese and sells the cheese to competing grocery store retailers for the same wholesale price. The two grocery stores then sell the cheese to customers under different brand names. This example is noteworthy because numerous grocery stores sell their own store brand products. The preceding example will be used throughout this paper to appreciate the linear and nonlinear demand models. An additional example pertains to a supply chain for sports equipment. Suppose a sports equipment manufacturer produces two different brands of tennis balls. The manufacturer sells one brand of tennis ball to one retailer and the other brand of tennis ball to the other retailer. The wholesale price is the same for both tennis balls. These are just two of a wide variety of examples that are appropriate for the supply chain model presented in this paper. In addition to differences in the products themselves, products can be differentiated by the location from which the product is sold, the speed of delivery, and the service provided [Friedman (1983)].

This paper explores two demand curves. The linear demand curve follows that of Singh & Vives (1984) and Varian (1992). The nonlinear demand curve is modeled after Shugan (1989) and Choi (1991). This paper utilizes the sufficient conditions stated in Friedman (1977) and Friedman (1983) to show existence and uniqueness of Cournot equilibrium. Existence and uniqueness of Cournot equilibrium is explicitly shown for the linear demand model. As for the

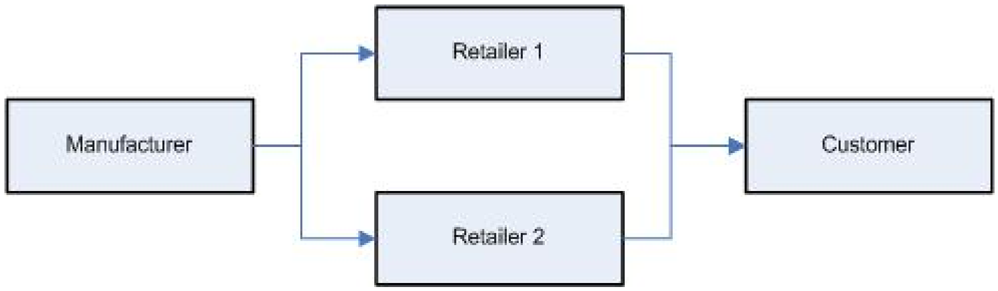

Figure 1.1 Oligopoly: One Manufacturer, Two Retailers

nonlinear demand model, existence and uniqueness of Cournot equilibrium is verified through computational efforts. Furthermore, the effects of initial demand, price effect, manufacturing variable cost, and cross price effect for a linear demand model and market potential, price elasticity of demand, manufacturing variable cost, and cross price elasticity of demand for a nonlinear demand model are investigated through sensitivity analysis. Conclusions of the analysis help the manufacturer and retailers determine which products to produce or sell.

Game theory and supply chain models have been combined in previous marketing and economic literature. Jeuland & Shugan (1983) studied the coordination issues that arise in a vertical supply chain consisting of one manufacturer and one retailer. McGuire & Staelin (1983) analyzed the consequences of vertical integration versus decentralization in a supply chain with two manufacturers and one retailer. Choi (1991) studied the equilibrium prices and profits of two manufacturers and one retailer acting under three different competitive behaviors: manufacturer-Stackelberg, retailer-Stackelberg, and Cournot. In addition, Yang & Zhou (2006) analyzed the optimal prices and profits of one manufacturer and two retailers playing in Cournot, collusion, and Stackelberg games.

Several supply chain models have examined differentiated products, an assumption which greatly influences the results of the model. McGuire & Staelin (1983) considered the effect of product substitutability in a duopoly of manufacturers that sell goods to a single retailer. The paper showed that if the manufacturers' products are highly substitutable in demand, decentralization is a Nash equilibrium. Conversely, if the products have low substitutability, each manufacturer will attain vertical integration. Moorthy (1988) extended on the research of McGuire & Staelin (1983) to identify the conditions under which decentralization is a Nash equilibrium. Under a linear demand curve, Choi (1991) showed that both wholesale and retail prices increase with product substitutability. In contrast, under a nonlinear demand curve, the opposite was true. As products became more substitutable, wholesale and retail prices decreased. Yang & Zhou (2006) studied the degree of substitutability between duopolistic retailers and showed that as substitutability increases, or the stronger the retailers compete, the higher the wholesale and retail prices.

A supply chain with a linear demand curve is the first to be presented in this paper. Among the papers to study linear demand and the supply chain include McGuire & Staelin (1983), Moorthy (1988), Choi (1991), and Yang & Zhou (2006). Linear demand curves set the foundation for research pertaining to other demand functions. The second demand model studied in this paper is a constant elasticity nonlinear demand function. The constant elasticity nonlinear demand function has many applications in theoretical and empirical research, and Shugan (1989) expands on its importance. Shugan used this particular demand function to explore the significance of product assortment by producers of economical, regular, and premium products. Choi (1991) also utilized a constant elasticity nonlinear demand function, which built on the findings of Choi's linear demand model.

The choice of demand curve is very important when modeling supply chains. Lau & Lau (2003) examined the effect several different demand curves have on decisions made in one-echelon, two-echelon, and three-echelon structures. The paper showed that equilibrium solutions in multi-echelon models change considerably over a small change in the demand curve. Consequently, economic differences between manufacturer and retailer profits result. Lau & Lau (2005) extended on Lau & Lau (2003) to show that results obtained with the assumption of a specific demand curve cannot be generalized to situations where other demand curves apply.

Existence and uniqueness of Cournot equilibrium for each model in this paper is presented. As described in Cachon & Netessine (2004), existence and uniqueness of equilibrium is very important in game theory. With existence of equilibrium, a solution to the game can be found. Once existence of equilibrium is established, uniqueness of equilibrium may or may not be achieved. A game with a unique equilibrium is significant because only one outcome will occur. Players can then have confidence that their decision will produce an equilibrium and the outcome that they expect.

The method used to determine existence of Cournot equilibrium in this paper reflects that of Friedman (1977) and Friedman (1983). In addition to several assumptions, the sufficient conditions for existence are: (1) the players' reaction functions intersect, (2) all players' assumptions are true, and (3) the second order sufficient condition (SOSC) is met. Friedman (1977) and Friedman (1983), in addition to Cachon & Netessine (2004) and Gallego, Huh, Kang, & Phillips (2006), use the contraction mapping approach to show uniqueness of equilibrium.

This paper makes contributions to the literature in three ways. First, it extends on the literature by modeling an oligopoly of one manufacturer and two retailers selling differentiated products and acting in Cournot competition. Second, unlike previous literature such as Choi (1991) and Yang & Zhou (2006), this paper demonstrates the sufficient conditions for existence and uniqueness of Cournot equilibrium in a two-echelon supply chain with linear and nonlinear demand. Third, this paper has three main findings: (1) the retailer with the lower price effect or lower price elasticity of demand has a higher margin, price, quantity demanded, and profit; (2) all players in the supply chain prefer to sell products with a high cross price effect or high cross price elasticity of demand; and (3) the manufacturer will always earn the most profit among the three members of the supply chain.

This paper is organized as follows. The next chapter develops the Cournot model with linear demand. The sufficient conditions for existence and uniqueness of Cournot equilibrium is presented. Chapter 3 summarizes the results of the linear demand model. Sensitivity analysis is performed to discuss the implications of initial demand, price effect, manufacturing variable cost, and product differentiation. Chapter 4 derives an alternative to the linear demand model by means of a nonlinear demand function. The sufficient conditions for existence and uniqueness of Cournot equilibrium is shown. Chapter 5 discusses the results of the nonlinear demand model. It is here that absolute market potential, price elasticity of demand, manufacturing variable cost, and cross price elasticity of demand are investigated through sensitivity analysis. The last chapter summarizes the important findings of this paper and proposes areas of future research.

CHAPTER 2. COURNOT MODEL WITH LINEAR DEMAND

This section begins with the assumptions and notation used in the Cournot model with linear demand. The derivation of the model is then performed, which includes the equilibrium margin, price, and quantity decisions. The sufficient conditions for existence and uniqueness of Cournot equilibrium is also shown.

2.1 Model Assumptions and Notation

This paper considers a two-echelon supply chain with one manufacturer and two retailers acting in Cournot competition. The manufacturer and retailers react simultaneously by setting their margins or prices. The corresponding order quantity is then determined. There is no cooperation between any of the players and no player has influence over the decisions of any other player. The objective for each player is to maximize profit. Neither the manufacturer nor the retailers participate in the game when profits are negative.

The manufacturer produces two types of products, and one type of product is sold to each retailer for the same wholesale price. In addition, wholesale and retail prices are assumed to be strictly positive and retailers i and j sell substitutable products. Further assumptions and notation are explained below.

m_i: retail margin per unit for retailer i, i=1,2;

$\hat{m}$: manufacturer margin per unit;

p_i: price per unit charged to the customers by retailer i, i=1,2;

w: wholesale price per unit charged to the retailers by the manufacturer;

c: manufacturing variable cost per unit;

Π_{R_i}: retailer i's profit, i=1,2;

Π_M: manufacturer's profit;

Π_C: total channel profit;

RT: ratio of the manufacturer's profit to the total profit of the retailers.

The downward sloping linear demand function that captures product substitutability is given by

$$q_i = a_i - b_i p_i + \gamma p_j, \qquad i, j = 1, 2, \qquad j \neq i. \tag{2.1}$$

Since $p_i = m_i + w$, (2.1) becomes

$$q_i = a_i - b_i(m_i + w) + \gamma(m_j + w), \qquad i, j = 1, 2, \qquad j \neq i, \tag{2.2}$$

where

q_i: customer demand, in units, faced by retailer i or the quantity ordered, in units, from the manufacturer by retailer i, i=1,2;

a_i: initial demand, retailer i, i=1,2;

b_i: price effect, retailer i, i=1,2;

γ: cross price effect.

Price effect is the measure of sensitivity of retailer i's sales to changes in retailer i's price. When price effect is high, changes in retailer i's price will significantly change the quantity demanded for retailer i. Similarly, when price effect is low, changes in retailer i's price will have little influence on the quantity demanded for retailer i. As seen in Varian (1992), cross price effects are symmetric, which is required for a well-behaved consumer demand function. Symmetric cross price effects is an important condition for a representative consumer to exist [Anderson, de Palma, & Thisse (1992)]. When cross price effect is high, changes in retailer i's price will significantly change the quantity demanded for retailer j. Likewise, when cross price effect is low, changes in retailer i's price will have minor influence on the quantity demanded for retailer j.

The parameters in (2.1) and (2.2) are assumed to satisfy $a_i > 0$ and $b_i > \gamma > 0$ [Choi (1991)]. $a_i > 0$ because demand is positive when $p_i = 0$. $b_i > \gamma$ since retailer i's own price effect has a greater influence on the quantity demanded for retailer i than the cross price effect. $\gamma > 0$ to signify that the products are substitutes. $\gamma < 0$ represents products that are complements, and when $\gamma = 0$, products are independent or completely differentiated from one another. When $a_i = a_j$, $\frac{\gamma^2}{b_i b_j}$ defines the index of product differentiation. As this index approaches zero, the products become more differentiated. If $\frac{\gamma^2}{b_i b_j} = 0$, the products are independent of one another. As this index approaches one, the products become more substitutable. When $\frac{\gamma^2}{b_i b_j} = 1$, the products are perfect substitutes; there is a homogeneous market [Singh & Vives (1984)].

2.2 Model Derivation

The manufacturer's profit is given by

$$\Pi_M = (w - c)(q_i + q_j), \qquad i, j = 1, 2, \qquad j \neq i, \tag{2.3}$$

and each retailer's profit is

$$\Pi_{R_i} = m_i q_i, \qquad i = 1, 2. \tag{2.4}$$

The manufacturer maximizes its profit and determines its wholesale price based on the retail margin of both retailers; the manufacturer takes the retail margins as given. The variables taken as given are identified by a bar in subsequent equations. The objective function for the manufacturer using (2.2) and (2.3) is

$$\max_w \Pi_M = (w - c)[a_1 - b_1(\bar{m}_1 + w) + \gamma(\bar{m}_2 + w) + a_2 - b_2(\bar{m}_2 + w) + \gamma(\bar{m}_1 + w)]. \tag{2.5}$$

Each retailer maximizes its profit and decides upon its retail margin based on the wholesale price of the manufacturer and retail price of the competing retailer. To put it differently, retailer i takes the wholesale price of the manufacturer and retailer j's price as given. The objective function for each retailer using (2.2) and (2.4) is

$$\begin{aligned}\max_{m_1} \Pi_{R_1} &= m_1[a_1 - b_1(m_1 + \bar{w}) + \gamma(\bar{m}_2 + \bar{w})],\\ \max_{m_2} \Pi_{R_2} &= m_2[a_2 - b_2(m_2 + \bar{w}) + \gamma(\bar{m}_1 + \bar{w})]. \end{aligned} \tag{2.6}$$

The manufacturer's reaction function is determined using the first order necessary condition (FONC) of the manufacturer's objective function in (2.5). The reaction function for each retailer is found using the FONC of the retailer's objective function in (2.6). The reaction function represents the margin or price in which a player is maximizing profit given the margin or price of all other players. The FONC for the manufacturer is

$$\frac{\partial \Pi_M}{\partial w} = a_1 + a_2 - b_1(\bar{m}_1 + 2w) - b_2(\bar{m}_2 + 2w) + \gamma(\bar{m}_1 + \bar{m}_2 + 4w) + c(b_1 + b_2 - 2\gamma) = 0. \tag{2.7}$$

The FONC for the two retailers is

$$\begin{aligned}\frac{\partial \Pi_{R_1}}{\partial m_1} = a_1 - b_1(2m_1 + \bar{w}) + \gamma(\bar{m}_2 + \bar{w}) &= 0,\\ \frac{\partial \Pi_{R_2}}{\partial m_2} = a_2 - b_2(2m_2 + \bar{w}) + \gamma(\bar{m}_1 + \bar{w}) &= 0. \end{aligned} \tag{2.8}$$

Solving (2.7) and (2.8) simultaneously using three equations and three unknown decision variables becomes

$$\begin{aligned} w^* &= \frac{2b_1b_2(a_1 + a_2 + 2b_1c + 2b_2c - 4c\gamma) + c\gamma^2(2\gamma - b_1 - b_2) + \gamma(a_1b_2 + a_2b_1)}{2(b_1 + b_2)(3b_1b_2 - \gamma^2) - 2\gamma(5b_1b_2 - \gamma^2)},\\ {m_1}^* &= \frac{a_2(\gamma - b_2)(2\gamma - b_1) + 2b_1b_2c(b_1 + b_2) - a_1b_2(2b_1 + 3b_2) + 5b_2\gamma(a_1 - b_1c) + c\gamma(b_2 + \gamma)(2\gamma - b_2) - b_1c\gamma^2}{2\gamma(5b_1b_2 - \gamma^2) - 2(b_1 + b_2)(3b_1b_2 - \gamma^2)},\\ {m_2}^* &= \frac{a_1(\gamma - b_1)(2\gamma - b_2) + 2b_1b_2c(b_1 + b_2) - a_2b_1(3b_1 + 2b_2) + 5b_1\gamma(a_2 - b_2c) + c\gamma(b_1 + \gamma)(2\gamma - b_1) - b_2c\gamma^2}{2\gamma(5b_1b_2 - \gamma^2) - 2(b_1 + b_2)(3b_1b_2 - \gamma^2)}. \end{aligned} \tag{2.9}$$

(2.9) represents an equilibrium for the decisions made by one manufacturer and two retailers and can be expressed by the vector $\begin{bmatrix} w^* & {m_1}^* & {m_2}^* \end{bmatrix}$. The Cournot equilibrium price vector can then be described as $\begin{bmatrix} w^* & {m_1}^* + w^* & {m_2}^* + w^* \end{bmatrix} = \begin{bmatrix} w^* & {p_1}^* & {p_2}^* \end{bmatrix}$. These are the price decisions that attain maximum profit for the manufacturer and retailers. No entity in the oligopoly can increase its profit by choosing a price different from this price vector, and price equilibrium is achieved. Further explanation on the sufficient conditions for existence and uniqueness of Cournot equilibrium appears in Section 2.3.

The equilibrium price and quantity is simplified when all parameters for retailer 1 and retailer 2 are equal: $a_1 = a_2 = a$ and $b_1 = b_2 = b$. Using (2.9), equilibrium wholesale price and retail margins are

$$\begin{aligned} w^* &= \frac{ab + c(2b-\gamma)(b-\gamma)}{3b^2 - 4b\gamma + \gamma^2}, \\ {m_1}^* &= \frac{a - c(b-\gamma)}{3b-\gamma}, \\ {m_2}^* &= \frac{a - c(b-\gamma)}{3b-\gamma}, \end{aligned} \tag{2.10}$$

and using (2.2) and (2.10), equilibrium quantities are

$$\begin{aligned} {q_1}^* &= \frac{b[a - c(b-\gamma)]}{3b-\gamma}, \\ {q_2}^* &= \frac{b[a - c(b-\gamma)]}{3b-\gamma}. \end{aligned} \tag{2.11}$$

(2.10) and (2.11) show that retail margins and quantities are identical when $a_1 = a_2 = a$ and $b_1 = b_2 = b$. In addition, $q_1 = bm_1$ and $q_2 = bm_2$ when $a_1 = a_2 = a$ and $b_1 = b_2 = b$. Using (2.3), (2.4), (2.10), and (2.11), the profit equations become

$$\begin{aligned} \Pi^*_M &= \frac{2b^2[a - c(b-\gamma)]^2}{(3b-\gamma)(3b^2 - 4b\gamma + \gamma^2)}, \\ \Pi^*_{R_1} &= \frac{b[a - c(b-\gamma)]^2}{(3b-\gamma)^2}, \\ \Pi^*_{R_2} &= \frac{b[a - c(b-\gamma)]^2}{(3b-\gamma)^2}. \end{aligned} \tag{2.12}$$

(2.12) shows that retailers have identical profits when $a_1 = a_2 = a$ and $b_1 = b_2 = b$. Also from (2.12), $RT = \frac{b}{b-\gamma}$. So, the manufacturer will always earn more profit than the combined profit of the two retailers when $a_1 = a_2 = a$ and $b_1 = b_2 = b$ since $a > 0$ and $b > \gamma > 0$.

2.3 Existence and Uniqueness of Cournot Equilibrium

2.3.1 Existence of Cournot Equilibrium

In order to show existence of Cournot equilibrium, several assumptions are made for the demand and cost functions. The conditions listed below are taken from Friedman (1977).

Assumption 1: $\forall p \geq 0$, demand is defined, continuous, bounded, and nonnegative.

Assumption 2: $\forall p > 0$, the demand function is twice continuously differentiable.

Assumption 3: Cost is defined and continuous for nonnegative demand. When demand equals zero, cost is greater than or equal to zero.

Assumption 4: The cost function is twice continuously differentiable for demand greater than zero.

In addition to these four assumptions, three conditions are sufficient for the existence of Cournot equilibrium and are found in Friedman (1977) and Friedman (1983). The three conditions are as follows: (1) the players' reaction functions intersect; (2) all players' assumptions regarding other players' actions turn out to be true; and (3) the second order sufficient condition (SOSC) is met. The SOSC ensures that the equilibrium is one that maximizes profit. Each condition for the existence of Cournot equilibrium will be shown as it relates to the linear demand model. Refer to Appendix A for an additional example illustrating the three sufficient conditions for existence of Cournot equilibrium.

2.3.1.1 Intersection of Players' Reaction Functions

The first sufficient condition to be presented for the existence of Cournot equilibrium is the intersection of the three players' reaction functions. The reaction functions for the manufacturer and retailers are found by solving (2.7) and (2.8) individually for each decision variable. The three reaction functions are

$$\begin{aligned} w(\bar{m}_1, \bar{m}_2) &= \frac{a_1 + a_2 - \bar{m}_1(b_1 - \gamma) - \bar{m}_2(b_2 - \gamma) + c(b_1 + b_2 - 2\gamma)}{2(b_1 + b_2 - 2\gamma)}, \\ m_1(\bar{w}, \bar{m}_2) &= \frac{a_1 - b_1\bar{w} + \gamma(\bar{m}_2 + \bar{w})}{2b_1}, \\ m_2(\bar{w}, \bar{m}_1) &= \frac{a_2 - b_2\bar{w} + \gamma(\bar{m}_1 + \bar{w})}{2b_2}. \end{aligned} \tag{2.13}$$

(2.9) is the intersection of the three reaction functions in (2.13).

2.3.1.2 Players' Assumptions are True

The previous section displayed the reaction functions of each firm, given each firm's beliefs about the other firms' actions. For Cournot equilibrium to exist, the assumptions each firm makes about another firm's actions must be the actual behavior. This can be described symbolically as $\bar{w} = w^*$, $\bar{m}_1 = m_1{}^*$, and $\bar{m}_2 = m_2{}^*$. If each firm actually knew the true reactions of every other firm, no firm would have incentive to deviate from its decision.

2.3.1.3 Second Order Sufficient Condition

The second order sufficient condition (SOSC) is the remaining sufficient condition for the existence of Cournot equilibrium. When the profit functions of the oligopoly members are concave at equilibrium, the SOSC is satisfied. According to Cachon & Netessine (2004), a concave profit function implies a unique reaction function but does not imply a unique equilibrium. Refer to Appendix A for an example that includes concave profit functions and multiple equilibria.

(2.14) shows that the SOSC is satisfied for the linear demand model. Each firm maximizes its profit since $b_1 > \gamma > 0$ and $b_2 > \gamma > 0$

$$\begin{aligned} \frac{\partial^2 \Pi_M}{\partial w^2} &= 4\gamma - 2(b_1 + b_2) < 0, \\ \frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} &= -2b_1 < 0, \\ \frac{\partial^2 \Pi_{R_2}}{\partial m_2^2} &= -2b_2 < 0. \end{aligned} \tag{2.14}$$

2.3.2 Uniqueness of Cournot Equilibrium

Uniqueness of Cournot equilibrium will be shown using the method presented in Friedman (1977), Friedman (1983), Cachon & Netessine (2004), and Gallego, Huh, Kang, & Phillips (2006). Each paper uses the contraction mapping argument to demonstrate uniqueness of equilibrium. In particular, Friedman (1983) uses this argument to show uniqueness of Cournot equilibrium with differentiated products. An example of a game with multiple equilibria is presented in Appendix A. Since the game has multiple equilibria, the contraction mapping argument fails, and this is shown in Appendix B.

In order for the contraction mapping argument to be satisfied, the reaction function of each firm has to be shown to be a contraction. Using the reaction functions from (2.13), the condition for uniqueness of Cournot equilibrium using the contraction mapping approach is

$$\begin{aligned} \left|\frac{\partial w(\bar{m}_1,\bar{m}_2)}{\partial \bar{m}_1}\right| &+ \left|\frac{\partial w(\bar{m}_1,\bar{m}_2)}{\partial \bar{m}_2}\right| < 1, \\ \left|\frac{\partial m_1(\bar{w},\bar{m}_2)}{\partial \bar{w}}\right| &+ \left|\frac{\partial m_1(\bar{w},\bar{m}_2)}{\partial \bar{m}_2}\right| < 1, \\ \left|\frac{\partial m_2(\bar{w},\bar{m}_1)}{\partial \bar{w}}\right| &+ \left|\frac{\partial m_2(\bar{w},\bar{m}_1)}{\partial \bar{m}_1}\right| < 1. \end{aligned} \tag{2.15}$$

This becomes

$$\begin{aligned} \frac{1}{2}\left|\frac{b_1-\gamma}{b_1+b_2-2\gamma}\right| + \frac{1}{2}\left|\frac{b_2-\gamma}{b_1+b_2-2\gamma}\right| &< 1, \\ \left|\frac{\gamma-b_1}{2b_1}\right| + \left|\frac{\gamma}{2b_1}\right| &< 1, \\ \left|\frac{\gamma-b_2}{2b_2}\right| + \left|\frac{\gamma}{2b_2}\right| &< 1. \end{aligned} \tag{2.16}$$

Each reaction function is a contraction since $b_1 > \gamma > 0$ and $b_2 > \gamma > 0$. Thus, uniqueness of Cournot equilibrium has been shown. There is only vector, namely $\begin{bmatrix} w^* & {m_1}^* & {m_2}^* \end{bmatrix}$ from (2.9), that satisfies the three reaction functions in (2.13).

CHAPTER 3. SENSITIVITY ANALYSIS OF LINEAR DEMAND

This chapter summarizes the results of the linear demand model. Sensitivity analysis was performed to determine the effects that four parameters had on equilibrium margins, prices, quantities, and profits. The parameters, which include initial demand, price effect, manufacturing variable cost, and cross price effect, lead the discussion for the analysis of the linear demand model. Observations related to the sensitivity analysis will be expressed, in addition to the economic and managerial implications for the manufacturer, retailers, and consumers.

The basic parameter values are $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$, and $\gamma = 2.0$. When the parameters c and γ were investigated, the parameters were allowed to vary between the respective upper and lower feasible limits: $w > c > 0$, $b_1 > \gamma > 0$, and $b_2 > \gamma > 0$. Additional parameters were allowed to differ by ±10%, ±20%, and ±50%, as seen in Arcelus & Srinivasan (1989). Using these parameter values and equations (2.2), (2.5), (2.6), and (2.9), the equilibrium quantities, profits, margins, and prices were computed.

The examples used for the sensitivity analysis are shown in Appendix C. Equilibrium decisions for the two-echelon model, total channel profit, and RT, the ratio of the manufacturer's profit to the total profit of the retailers, will be addressed in the following sections.

3.1 Initial Demand

The results of the sensitivity analysis for initial demand is shown in Figures 3.1, 3.2, 3.3, and 3.4. The initial demand for retailer 1 was varied while all other parameters were held constant. The conclusions relating to retailer 1 are identical to the conclusions that would be made for retailer 2, had the initial demand for retailer 2 been altered instead of retailer 1.

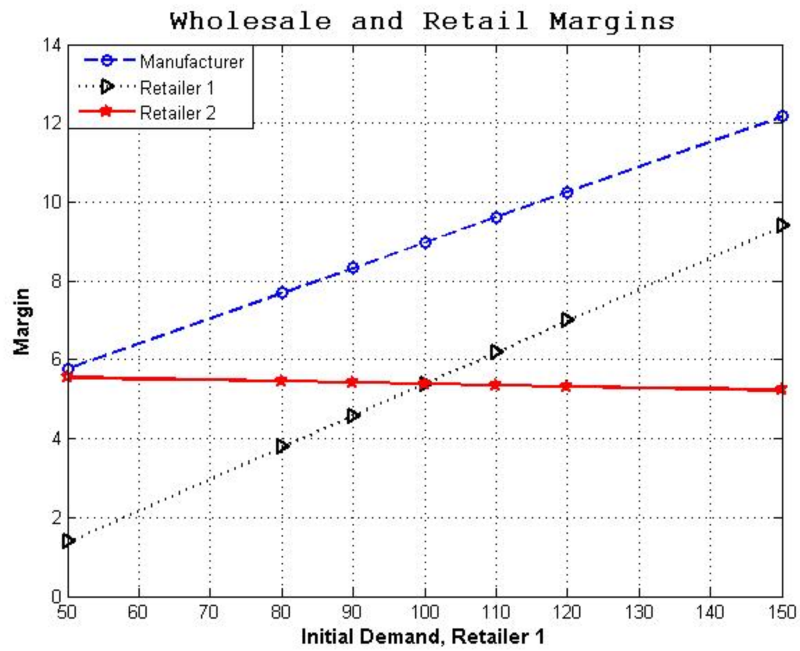

Figure 3.1 a_1: $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

As the initial demand for retailer 1 increases, margin for retailer 1 also increases, which leads to an increase in total profit for retailer 1. When initial demand is small, retail margin is low, but when initial demand is large, retail margin is high. The same is true for retail price. Retailer 1 sells its product at a low price when the initial demand for the product is small and at a high price when initial demand is large. Retailer 2 has decreasing retail margin, increasing retail price, and decreasing quantity demanded as the initial demand for retailer 1 increases. This set of conditions results in decreasing profit for retailer 2.

The manufacturer margin, wholesale price, and manufacturer profit increase with initial demand. Also, the manufacturer earns the largest profit among all members of the supply chain regardless of the initial demand for retailer 1.

Figure 3.3 shows that demand for retailer 1 increases with initial demand while the demand for retailer 2 slightly decreases. As displayed in Appendix C, the smaller the difference between initial demand for retailer 1 and initial demand for retailer 2, the higher RT becomes. RT is highest when the retailers have identical initial demands. In addition, total profit of the supply chain increases as initial demand for retailer 1 increases.

Figure 3.2 a_1: $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

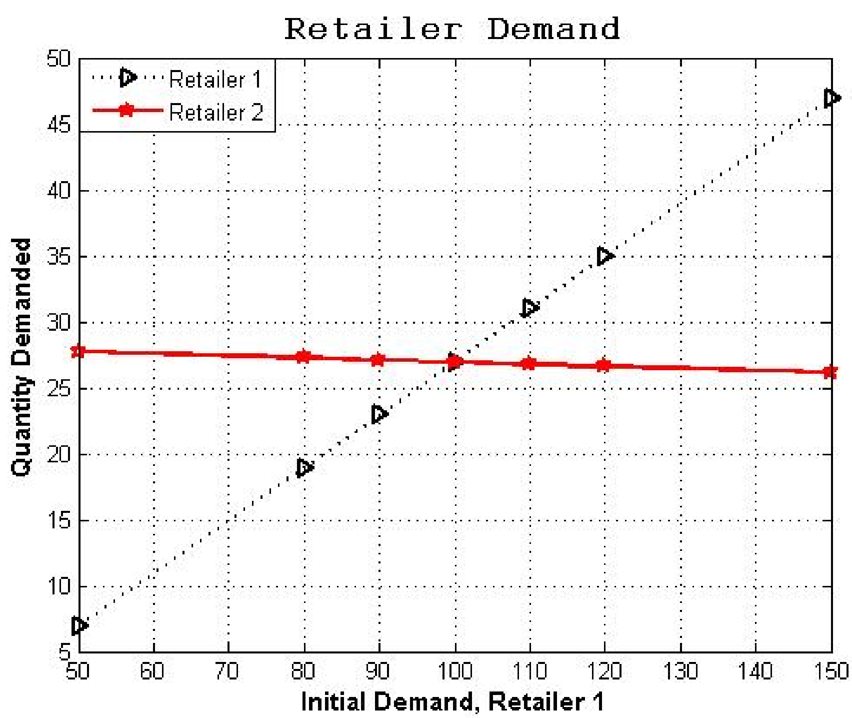

Figure 3.3 a_1: $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

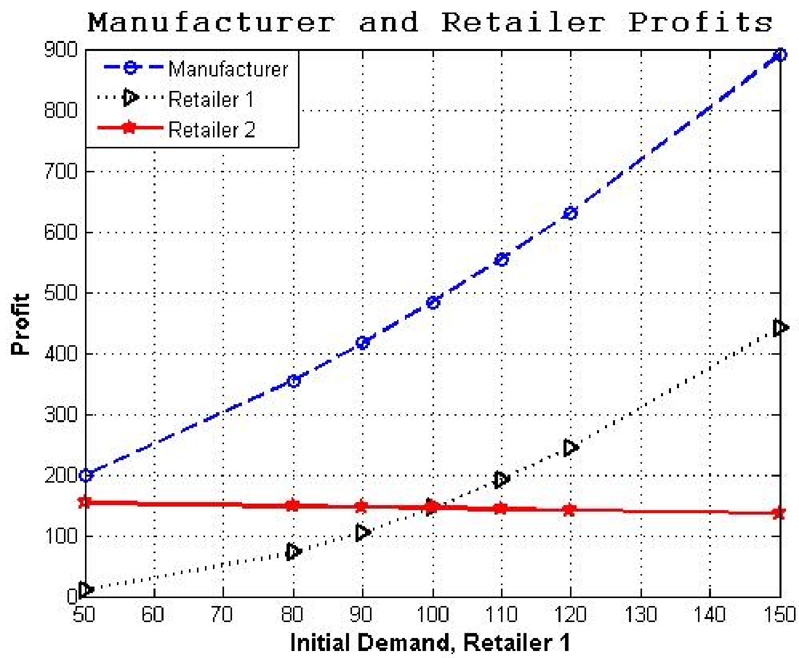

Figure 3.4 a_1: $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

The manufacturer prefers initial demand to be as large as possible since this is when the manufacturer earns the largest profit. The retailer with the larger initial demand has a higher retail margin, higher retail price, larger quantity demanded, and larger profit, *ceteris paribus*. Thus, large initial demand is beneficial to the entire supply chain. Using the cheese example from Chapter 1, if customers have a large initial demand for cheese, grocery store retailers will benefit from high sales. Additionally, the manufacturer will benefit by producing a large quantity of cheese for the grocery stores. From the point of view of the consumer, surplus is largest when initial demand is smallest. Consumers face a low retail price that is similar to wholesale price when initial demand is small.

3.2 Price Effect

Figures 3.5, 3.6, 3.7, and 3.8 display the sensitivity analysis in regards to price effect. Price effect for retailer 1 was allowed to vary while all other parameters were held constant. Had price effect for retailer 2 been analyzed, the conclusions for the two retailers would be reversed.

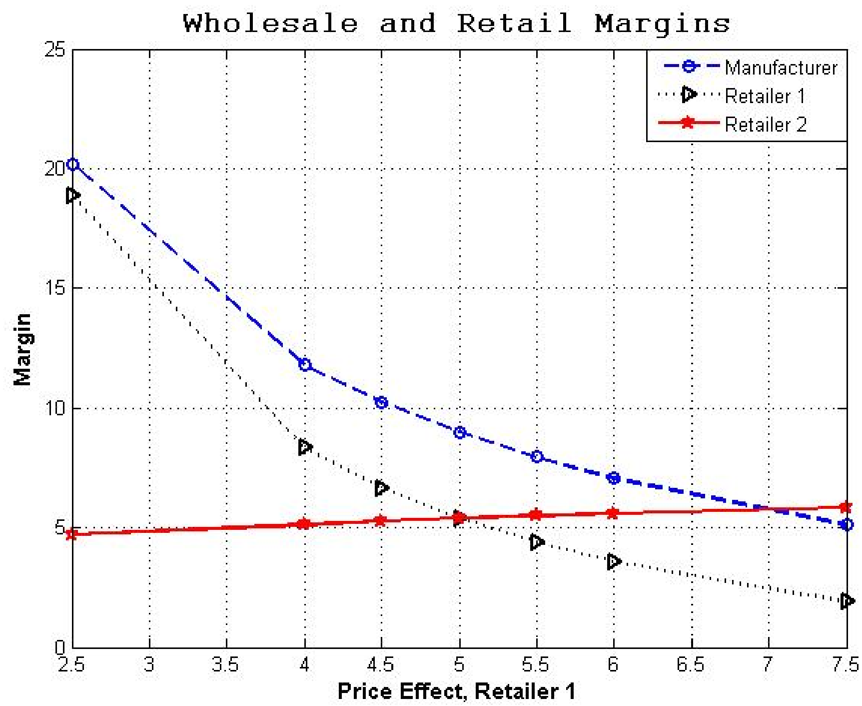

Figure 3.5 b_1: $a_1 = 100$, $a_2 = 100$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

Figures 3.5 and 3.6 show that as price effect for retailer 1 increases, both retail margin and retail price for retailer 1 decrease. When price effect is low, retail margin is high. When price effect is high, retail margin is low. Retail margin for retailer 2 slightly increases as price effect for retailer 1 increases. Retail price for retailer 2 and wholesale price for the manufacturer both decrease but decrease at a slower rate than the retail price for retailer 1. The manufacturer margin also decreases as price effect increases.

As price effect for retailer 1 increases in Figure 3.7, quantity demanded decreases for retailer 1 and increases for retailer 2. Price effect also influences the profits of the supply chain, as can be seen in Figure 3.8. When price effect for retailer 1 is low, retailer 1 has a large profit. When price effect for retailer 1 is high, retailer 1 has a small profit while retailer 2 has a profit comparable to the manufacturer. The manufacturer has the largest profit among the members of the oligopoly regardless of the price effect for retailer 1. The profits for retailer 1 and the manufacturer decrease as price effect for retailer 1 increases. Conversely, retailer 2 has increasing profits with increasing price effect for retailer 1.

Figure 3.6 b_1: $a_1 = 100$, $a_2 = 100$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

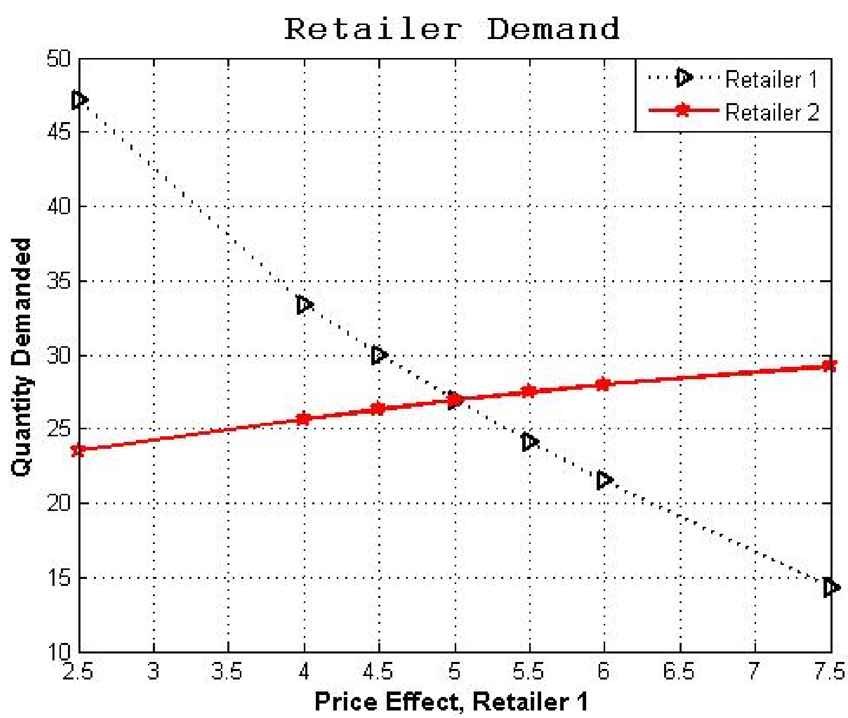

Figure 3.7 b_1: $a_1 = 100$, $a_2 = 100$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

Figure 3.8 b_1: $a_1 = 100$, $a_2 = 100$, $b_2 = 5.0$, $c = 10$, $\gamma = 2.0$

Further analysis is done using the examples in Appendix C. As the difference between the price effect for the retailers increases, RT generally decreases. Furthermore, total supply chain profit decreases as price effect increases.

RT is highest when the difference between the price effects for the retailers is small but not equal. The manufacturer achieves its largest profit when price effect is lowest. Based on these two observations, the manufacturer would benefit when price effect for the retailers is similar and as low as possible. The retailer with the lower price effect will have a higher retail margin, higher retail price, larger quantity demanded, and larger profit in relation to the other retailer. A retailer will benefit the most when its price effect is low and the difference between its own price effect and price effect for the other retailer is large. So, in regards to the latter, the manufacturer and retailers have conflicting objectives. Consumers benefit when price effect is high because they pay for a low retail price similar to that of the wholesale price. However, when price effect is low, consumers incur high retail prices.

Each member in the oligopoly benefits when price effect for the retailers is low. So, if the price effect for a product, such as cheese, is low, grocery store retailers would prefer to sell this product. However, if the price effect for a product is high, grocery store retailers may abandon the product and, in its place, sell a product with a lower price effect. The same idea applies to the manufacturer. If empirical studies have shown that the price effect for a product the manufacturer produces is high, the manufacturer may consider producing a product with a lower price effect.

3.3 Manufacturing Variable Cost

This section investigates the implications behind manufacturing variable cost. Only the values leading to a positive profit were examined ($w > c > 0$). Figures 3.9, 3.10, 3.11, and 3.12, along with the examples in Appendix C, help direct the discussion for this section.

Figure 3.9 c: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $\gamma = 2.0$

Figure 3.10 c: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $\gamma = 2.0$

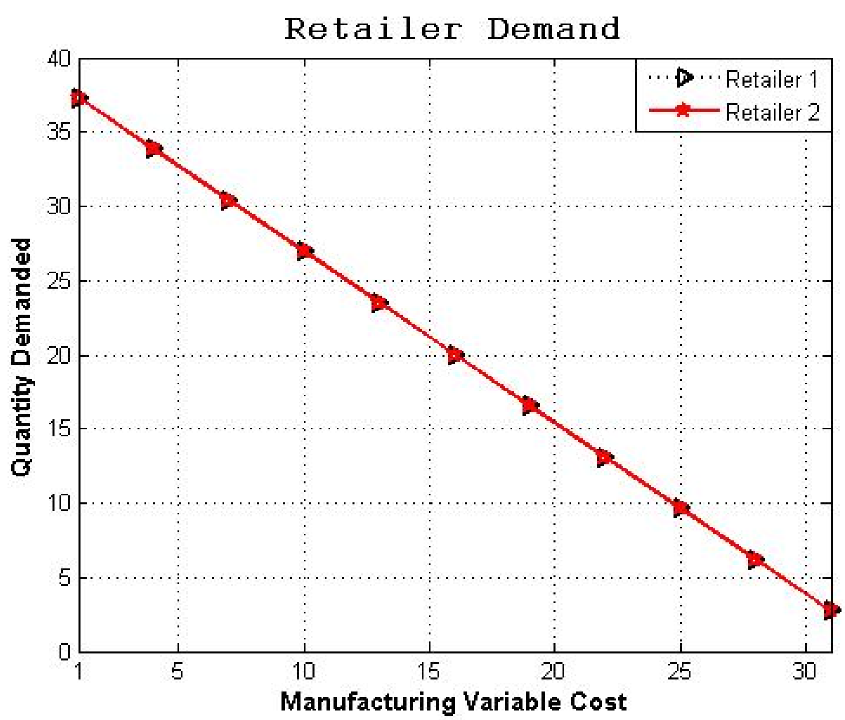

Figure 3.11 c: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $\gamma = 2.0$

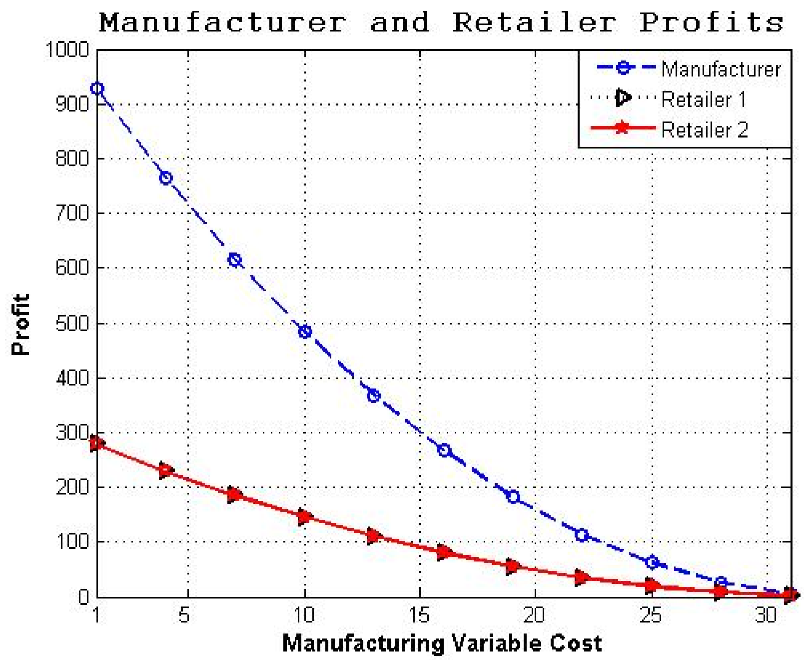

Figure 3.12 c: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $\gamma = 2.0$

Margins decrease and prices increase as manufacturing variable cost increases. Retail margins and prices are identical for both retailers regardless of manufacturing variable cost since $a_1 = a_2$ and $b_1 = b_2$; refer to equation (2.10). An increase in manufacturing variable cost leads to a smaller increase in wholesale price and an even smaller increase in retail price.

Quantity demanded decreases as manufacturing variable cost increases and is identical for both retailers since $a_1 = a_2$ and $b_1 = b_2$; refer to equation (2.11). Quantity demanded is large when manufacturing variable cost is small and is small when manufacturing variable cost is large.

The combination of decreasing manufacturer and retail margins and decreasing demand results in decreasing profits. Conversely, the manufacturer and retailers have large profits when manufacturing variable cost is low. Manufacturing variable cost does not alter RT since initial demand and price effects are symmetric; refer to equation (2.12).

The oligopoly and consumers alike benefit from low manufacturing variable cost. The cheese example from Chapter 1 is used to demonstrate this concept. If the manufacturing variable cost for cheese is low, the manufacturer can produce cheese at a low cost; this benefits the manufacturer. The manufacturer is then able to sell cheese to the grocery store retailers at a low wholesale price; this benefits the grocery stores. Finally, due to the low wholesale price, grocery stores are able to sell cheese to the consumers at a low retail price; this benefits the consumers.

3.4 Cross Price Effect

The final parameter to be analyzed for the linear demand model is cross price effect. Cross price effect has been varied between the upper and lower limits for this analysis, $b_1 > \gamma > 0$ and $b_2 > \gamma > 0$, while all other parameters were held constant. Referring back to Chapter 2, $\frac{\gamma^2}{b_1 b_2}$ defines the index of product differentiation when $a_1 = a_2$. Since cross price effect is the only parameter allowed to change in this analysis, an increase in cross price effect is equivalent to stating that products are more substitutable or less differentiated. Likewise, a decrease in cross price effect is equivalent to stating that products are less substitutable or more differentiated.

Figure 3.13 γ: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$

Figure 3.14 γ: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$

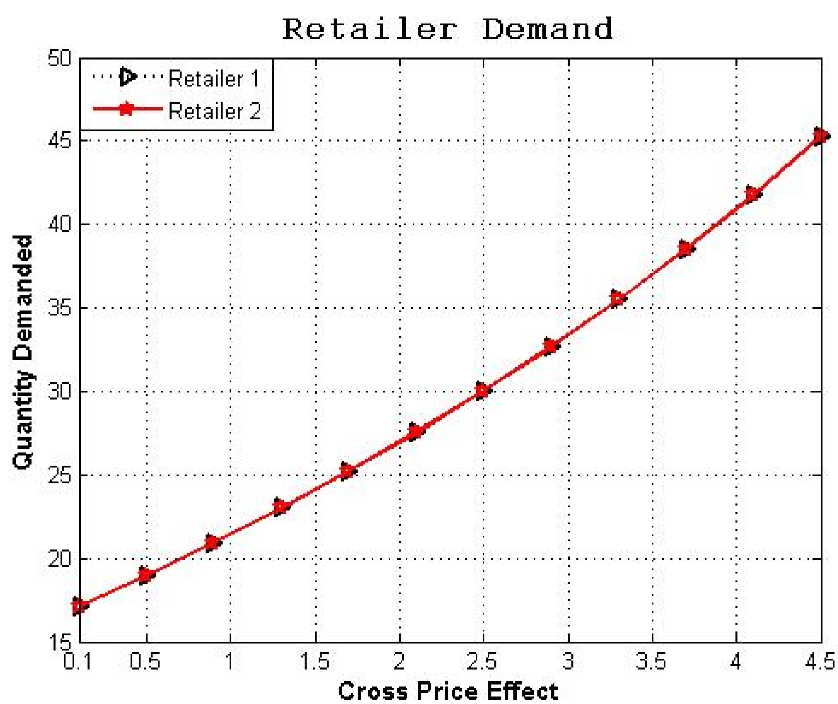

Figure 3.15 γ: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$

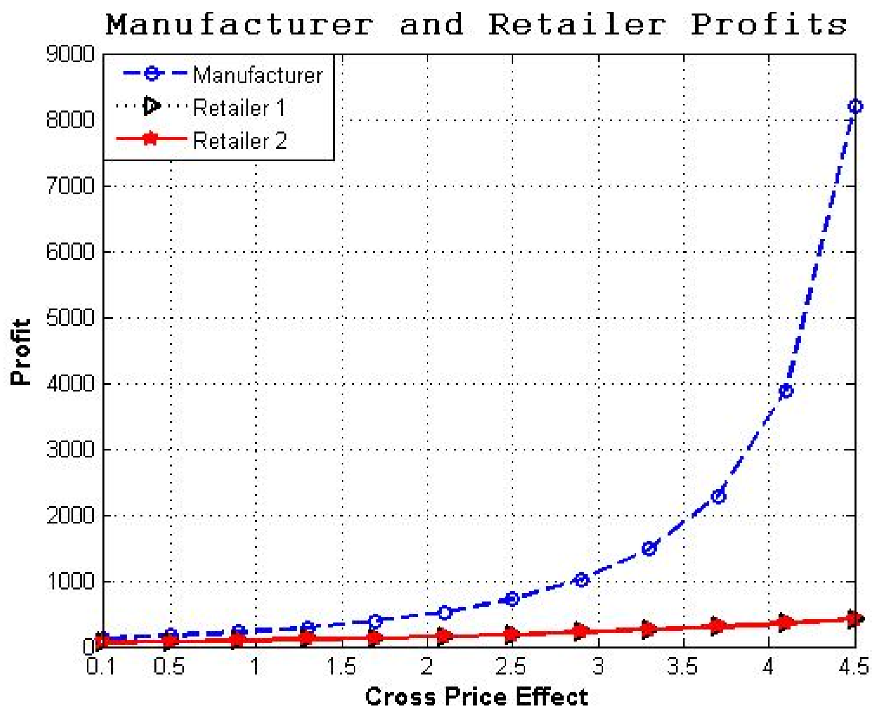

Figure 3.16 γ: $a_1 = 100$, $a_2 = 100$, $b_1 = 5.0$, $b_2 = 5.0$, $c = 10$

As cross price effect increases, margins, prices, quantity demanded, and profits of all supply chain players increase. Retailers have identical margins, prices, quantity demanded, and profits regardless of cross price effect since $a_1 = a_2$ and $b_1 = b_2$. Equations (2.10), (2.11), and (2.12) support these observations. Furthermore, margins, prices, quantity demanded, and profits are highest when goods are close substitutes ($\frac{\gamma^2}{b_1 b_2} \to 1$). The opposite is true when goods are highly differentiated ($\frac{\gamma^2}{b_1 b_2} \to 0$). RT increases as cross price effect increases and is highest when products are close substitutes. Figures 3.13, 3.14, 3.15, and 3.16, along with the examples in Appendix C, reflect the previous observations.

When products are close substitutes, the manufacturer has a large profit due to high manufacturer margin and large demand. Wholesale price increases as cross price effect increases even though manufacturing variable cost stays constant. Similarly, retailers have high margins, high prices, large demand, and large profits when cross price effect is high. Since the objective for the manufacturer and retailers is to maximize profit, the manufacturer benefits from producing products that have a high cross price effect, and retailers benefit from selling products that have a high cross price effect. Furthermore, each retailer may consider selling several substitutable products, such as multiple brands of cheese, to earn more profit.

Consumers benefit in two ways from a low cross price effect. First, retail prices are low. Second, consumers can buy differentiated products. Conversely, consumers endure high retail prices and have a limited product selection when cross price effect is high.

The sensitivity analysis for cross price effect has brought about a weakness in the linear demand model. As products become more substitutable, prices increase. This is contrary to intuition, but similar conclusions were made by Choi (1991) and Yang & Zhou (2006). Other papers, such as Choi (1996), have found a way around this weakness of the linear demand model, but improbable assumptions were made.

CHAPTER 4. COURNOT MODEL WITH NONLINEAR DEMAND

This chapter follows a similar outline as that of Chapter 2 but with a constant elasticity nonlinear demand function. Derivation is carried out, which is then followed by the sufficient conditions for existence and uniqueness of Cournot equilibrium.

4.1 Model Assumptions and Notation

The manufacturer produces two types of products, and one type of product is sold to each retailer for the same wholesale price. The objective for the manufacturer and each retailer is to maximize profit, and no player operates when profits are negative. In addition, wholesale and retail prices are assumed to be strictly positive, and retailers i and j sell substitutable products. Further assumptions and notation are given below.

m_i: retail margin per unit for retailer i, i=1,2;

$\hat{m}$: manufacturer margin per unit;

p_i: price per unit charged to the customers by retailer i, i=1,2;

w: wholesale price per unit charged to the retailers by the manufacturer;

c: manufacturing variable cost per unit;

Π_{R_i}: retailer i's profit, i=1,2;

Π_M: manufacturer's profit;

Π_C: total channel profit;

RT: ratio of the manufacturer's profit to the total profit of the retailers.

The downward sloping constant elasticity nonlinear demand function that captures differentiated products is given by

$$q_i = \alpha_i (p_i)^{-\beta_i} (p_j)^{\delta_i}, \qquad i, j = 1, 2, \qquad j \neq i. \tag{4.1}$$

Since $p_i = m_i + w$, (4.1) becomes

$$q_i = \alpha_i (m_i + w)^{-\beta_i} (m_j + w)^{\delta_i}, \qquad i, j = 1, 2, \qquad j \neq i, \tag{4.2}$$

where

q_i: customer demand, in units, faced by retailer i or the quantity ordered, in units, from the manufacturer by retailer i, i=1,2;

α_i: a measure of retailer i's absolute market potential, i=1,2;

β_i: price elasticity of demand, retailer i, i=1,2;

δ_i: cross price elasticity of demand, retailer i, i=1,2.

The price elasticity of demand is a constant value equal to that of β_i. If β_i is high, the quantity demanded for retailer i significantly changes as the price for retailer i changes. If β_i is low, the quantity demanded for retailer i stays relatively the same as the price for retailer i changes. The cross price elasticity of demand is a constant value equal to that of δ_i . If δ_i is high, the quantity demanded for retailer i drastically changes when the price for retailer j changes. If δ_i is low, the quantity demanded for retailer i stays relatively the same when the price for retailer j changes.

The parameters in (4.1) and (4.2) are assumed to satisfy $\alpha_i > 0$, $\beta_i > 2$, and $1 > \delta_i > 0$. $\alpha_i > 0$ because market potential is positive. For equilibrium to exist, $\beta_i > 2$. As a result, the price elasticity of demand will always be elastic. Finally, $1 > \delta_i > 0$ to model substitute products. When $\delta_i < 0$, products are complements.

4.2 Model Derivation

The manufacturer's profit is given by equation (2.3). After expanding the manufacturer's profit equation to include equation (4.2), the manufacturer's objective function becomes

$$\max_{w} \Pi_M = (w-c)[\alpha_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1}+\alpha_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2}]. \tag{4.3}$$

The manufacturer determines its wholesale price based on each retailer's retail margin; the manufacturer takes the retail margins as given. The variables taken as given are identified by a bar in equation (4.3). The same is true for subsequent equations. Each retailer bases its retail price on the wholesale price of the manufacturer and retail price of the competing retailer; each retailer takes these values as given.

The profit for each retailer is given by equation (2.4). The objective function for each retailer using (2.4) and (4.2) is

$$\begin{aligned}
\max_{m_1} \Pi_{R_1} &= m_1\alpha_1(m_1+\bar{w})^{-\beta_1}(\bar{m}_2+\bar{w})^{\delta_1},\\
\max_{m_2} \Pi_{R_2} &= m_2\alpha_2(m_2+\bar{w})^{-\beta_2}(\bar{m}_1+\bar{w})^{\delta_2}.
\end{aligned} \tag{4.4}$$

The first order necessary condition (FONC) for the manufacturer is

$$\begin{aligned}
\frac{\partial \Pi_M}{\partial w} &= \alpha_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1}+\alpha_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2}\\
&- \alpha_1\beta_1 w(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1}+\alpha_1\delta_1 w(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-1}\\
&- \alpha_2\beta_2 w(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2}+\alpha_2\delta_2 w(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-1}\\
&+ \alpha_1\beta_1 c(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1}-\alpha_1 c\delta_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-1}\\
&+ \alpha_2\beta_2 c(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2}-\alpha_2 c\delta_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-1}=0.
\end{aligned} \tag{4.5}$$

The FONC for the two retailers is

$$\begin{aligned}
\frac{\partial \Pi_{R_1}}{\partial m_1} &= \alpha_1(\bar{m}_2+\bar{w})^{\delta_1}[(m_1+\bar{w})^{-\beta_1}-\beta_1 m_1(m_1+\bar{w})^{-\beta_1-1}]=0,\\
\frac{\partial \Pi_{R_2}}{\partial m_2} &= \alpha_2(\bar{m}_1+\bar{w})^{\delta_2}[(m_2+\bar{w})^{-\beta_2}-\beta_2 m_2(m_2+\bar{w})^{-\beta_2-1}]=0.
\end{aligned} \tag{4.6}$$

Using equation (4.6), the reaction functions for the retailers take the form of

$$\begin{aligned} m_1(\bar{w}, \bar{m}_2) &= \frac{w}{\beta_1 - 1}, \\ m_2(\bar{w}, \bar{m}_1) &= \frac{w}{\beta_2 - 1}. \end{aligned} \tag{4.7}$$

As (4.7) shows, the retail margin for retailer i is independent of the retail margin for retailer j. Furthermore, retail margins are identical when $\beta_1 = \beta_2 = \beta$. The reaction function for the manufacturer cannot be stated as an explicit function. So, it will be described by the implicit function in (4.5).

A closed form solution for the equilibrium is possible when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$. Through the use of (4.5) and (4.7), the three unknown decision variables become

$$\begin{aligned} w^* &= \frac{c(\beta - 1)(\beta - \delta)}{\beta^2 - 2\beta - \beta\delta + \delta}, \\ {m_1}^* &= \frac{c(\beta - \delta)}{\beta^2 - 2\beta - \beta\delta + \delta}, \\ {m_2}^* &= \frac{c(\beta - \delta)}{\beta^2 - 2\beta - \beta\delta + \delta}. \end{aligned} \tag{4.8}$$

(4.8) represents the equilibrium wholesale price and retail margins and is the intersection of the reaction functions for the manufacturer and two retailers when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$. The equilibrium depends on three parameters, namely manufacturing variable cost, price elasticity of demand, and cross price elasticity of demand. (4.8) can be expressed by the vector $\begin{bmatrix} w^* & {m_1}^* & {m_2}^* \end{bmatrix}$. The Cournot equilibrium price vector can then be described as $\begin{bmatrix} w^* & {m_1}^* + w^* & {m_2}^* + w^* \end{bmatrix} = \begin{bmatrix} w^* & {p_1}^* & {p_2}^* \end{bmatrix}$. No entity in the oligopoly can increase its profit by making a decision different from this Cournot equilibrium price vector.

Equilibrium quantities are determined using (4.2) and (4.8) and are represented by

$$\begin{aligned} {q_1}^* &= \alpha\left(\frac{\beta c(\beta - \delta)}{\beta^2 - 2\beta - \beta\delta + \delta}\right)^{\delta - \beta}, \\ {q_2}^* &= \alpha\left(\frac{\beta c(\beta - \delta)}{\beta^2 - 2\beta - \beta\delta + \delta}\right)^{\delta - \beta}. \end{aligned} \tag{4.9}$$

As (4.8) and (4.9) show, retail margins and quantity demanded are identical when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$.

Using (2.3), (2.4), (4.8), and (4.9), the profit equations become

$$\begin{aligned}
\Pi_M^* &= \frac{2\alpha\beta c(\frac{\beta c(\beta-\delta)}{\beta^2-2\beta-\beta\delta+\delta})^{\delta-\beta}}{\beta^2-2\beta-\beta\delta+\delta}, \\
\Pi_{R_1}^* &= \frac{\alpha c(\beta-\delta)(\frac{\beta c(\beta-\delta)}{\beta^2-2\beta-\beta\delta+\delta})^{\delta-\beta}}{\beta^2-2\beta-\beta\delta+\delta}, \\
\Pi_{R_2}^* &= \frac{\alpha c(\beta-\delta)(\frac{\beta c(\beta-\delta)}{\beta^2-2\beta-\beta\delta+\delta})^{\delta-\beta}}{\beta^2-2\beta-\beta\delta+\delta}.
\end{aligned} \tag{4.10}$$

(4.10) shows retail profits are identical when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$. Also, unlike equilibrium prices, equilibrium quantities and profits are dependent on all parameters. Additionally, $RT = \frac{\beta}{\beta-\delta}$, which is derived from (4.10). Thus, the manufacturer will always earn more profit than the combined profit of the two retailers when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$ since $\alpha > 0$, $\beta > 2$, and $1 > \delta > 0$.

4.3 Existence and Uniqueness of Cournot Equilibrium

4.3.1 Existence of Cournot Equilibrium

The four assumptions and three sufficient conditions for the existence of Cournot equilibrium addressed in Chapter 2 also apply to the nonlinear demand model. Each of the three sufficient conditions will be shown in this section. Due to the complexity of the nonlinear demand model, existence of Cournot equilibrium is not explicitly stated but is determined computationally. The values used for the parameters of the computations are specified in Chapter 5 and listed in Appendix D.

4.3.1.1 Intersection of Players' Reaction Functions

The first sufficient condition to be presented for the existence of Cournot equilibrium is the intersection of the three players' reaction functions. The intersection of the manufacturer's implicit reaction function in (4.5) and the retailers' reaction functions in (4.7) was found through computational efforts. The equilibrium wholesale price and retail margins that resulted were then used to compute equilibrium quantities and profits. The intersection of the three reaction functions when $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $\delta_1 = \delta_2 = \delta$ is shown in (4.8).

4.3.1.2 Players' Assumptions are True

Again, this condition follows that of Chapter 2. For Cournot equilibrium to exist, the assumptions each firm has about another firm's actions must be the actual behavior. This is described symbolically as $\bar{w} = w^*$, $\bar{m}_1 = {m_1}^*$, and $\bar{m}_2 = {m_2}^*$.

4.3.1.3 Second Order Sufficient Condition

The second order sufficient condition (SOSC) is the remaining sufficient condition to establish existence of Cournot equilibrium. When each profit function is concave at equilibrium, the SOSC is satisfied. The condition is represented mathematically as

$$\begin{aligned}
\frac{\partial^2 \Pi_M}{\partial w^2} &< 0, \\
\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} &< 0, \\
\frac{\partial^2 \Pi_{R_2}}{\partial m_2^2} &< 0.
\end{aligned} \tag{4.11}$$

This becomes

$$\begin{aligned}
\frac{\partial^2 \Pi_M}{\partial w^2} &= \alpha_1\delta_1(\delta_1 - 1)(w-c)(\bar{m}_1 + w)^{-\beta_1}(\bar{m}_2 + w)^{\delta_1 - 2} \\
&+ \alpha_2\delta_2(\delta_2 - 1)(w-c)(\bar{m}_2 + w)^{-\beta_2}(\bar{m}_1 + w)^{\delta_2 - 2} \\
&+ \alpha_1\beta_1(\beta_1 + 1)(w-c)(\bar{m}_1 + w)^{-\beta_1 - 2}(\bar{m}_2 + w)^{\delta_1} \\
&+ \alpha_2\beta_2(\beta_2 + 1)(w-c)(\bar{m}_2 + w)^{-\beta_2 - 2}(\bar{m}_1 + w)^{\delta_2} \\
&- 2\alpha_1\beta_1\delta_1(w-c)(\bar{m}_1 + w)^{-\beta_1 - 1}(\bar{m}_2 + w)^{\delta_1 - 1} \\
&- 2\alpha_2\beta_2\delta_2(w-c)(\bar{m}_2 + w)^{-\beta_2 - 1}(\bar{m}_1 + w)^{\delta_2 - 1} \\
&- 2\alpha_1\beta_1(\bar{m}_1 + w)^{-\beta_1 - 1}(\bar{m}_2 + w)^{\delta_1} + 2\alpha_1\delta_1(\bar{m}_2 + w)^{\delta_1 - 1}(\bar{m}_1 + w)^{-\beta_1} \\
&- 2\alpha_2\beta_2(\bar{m}_2 + w)^{-\beta_2 - 1}(\bar{m}_1 + w)^{\delta_2} + 2\alpha_2\delta_2(\bar{m}_1 + w)^{\delta_2 - 1}(\bar{m}_2 + w)^{-\beta_2} < 0, \\
\frac{\partial^2 \Pi_{R_1}}{\partial m_1^2} &= \alpha_1\beta_1(\bar{m}_2 + \bar{w})^{\delta_1}[m_1(\beta_1 + 1)(m_1 + \bar{w})^{-\beta_1 - 2} - 2(m_1 + \bar{w})^{-\beta_1 - 1}] < 0, \\
\frac{\partial^2 \Pi_{R_2}}{\partial m_2^2} &= \alpha_2\beta_2(\bar{m}_1 + \bar{w})^{\delta_2}[m_2(\beta_2 + 1)(m_2 + \bar{w})^{-\beta_2 - 2} - 2(m_2 + \bar{w})^{-\beta_2 - 1}] < 0.
\end{aligned} \tag{4.12}$$

Due to the intricacy of (4.12), the SOSC for the manufacturer and two retailers was determined computationally. Each example used for the sensitivity analysis in Chapter 5 and shown in

Appendix D satisfies the SOSC.

4.3.2 Uniqueness of Cournot Equilibrium

Uniqueness of Cournot equilibrium will be shown using the contraction mapping argument. If the implicit reaction function for the manufacturer in (4.5) and the retailers' reaction functions in (4.7) are contractions, then there is a unique Cournot equilibrium. The condition for the retailers' reaction functions to be contractions is

$$\begin{aligned} \left|\frac{\partial m_1(\bar{w},\bar{m}_2)}{\partial \bar{w}}\right| &+ \left|\frac{\partial m_1(\bar{w},\bar{m}_2)}{\partial \bar{m}_2}\right| < 1, \\ \left|\frac{\partial m_2(\bar{w},\bar{m}_1)}{\partial \bar{w}}\right| &+ \left|\frac{\partial m_2(\bar{w},\bar{m}_1)}{\partial \bar{m}_1}\right| < 1. \end{aligned} \tag{4.13}$$

This becomes

$$\begin{aligned} \left|\frac{1}{\beta_1 - 1}\right| + |0| &< 1, \\ \left|\frac{1}{\beta_2 - 1}\right| + |0| &< 1. \end{aligned} \tag{4.14}$$

(4.14) shows the retailers' reaction functions are contractions since $\beta_1 > 2$ and $\beta_2 > 2$.

If the manufacturer had an explicit reaction function, the following condition would be sufficient to show that the manufacturer's reaction function is a contraction

$$\left|\frac{\partial w(\bar{m}_1,\bar{m}_2)}{\partial \bar{m}_1}\right| + \left|\frac{\partial w(\bar{m}_1,\bar{m}_2)}{\partial \bar{m}_2}\right| < 1. \tag{4.15}$$

(4.15) is equivalent to

$$\left|-\frac{\frac{\partial^2 \Pi_M}{\partial w \partial \bar{m}_1}}{\frac{\partial^2 \Pi_M}{\partial w^2}}\right| + \left|-\frac{\frac{\partial^2 \Pi_M}{\partial w \partial \bar{m}_2}}{\frac{\partial^2 \Pi_M}{\partial w^2}}\right| < 1. \tag{4.16}$$

Since the manufacturer has an implicit reaction function, the implicit function theorem, as described in Toumanoff & Nourzad (1994), is used to determine if the manufacturer's reaction function is a contraction. The condition in (4.16) is equivalent to the uniqueness of equilibrium condition specified in Friedman (1977). For the nonlinear demand model, (4.16) becomes

$$| \, [\alpha_1\beta_1\delta_1(c-w)(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1-1} - \alpha_1\beta_1(\beta_1+1)(c-w)(\bar{m}_1+w)^{-\beta_1-2}(\bar{m}_2+w)^{\delta_1}$$

$$- \; \alpha_1\beta_1(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1} - \alpha_2\delta_2(\delta_2-1)(c-w)(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-2}$$

$$
\begin{aligned}
+ &\quad \alpha_2\beta_2\delta_2(c-w)(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2-1} + \alpha_2\delta_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-1}] \\
/ &\quad [\alpha_1\delta_1(\delta_1-1)(w-c)(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-2} + \alpha_2\delta_2(\delta_2-1)(w-c)(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-2} \\
+ &\quad \alpha_1\beta_1(\beta_1+1)(w-c)(\bar{m}_1+w)^{-\beta_1-2}(\bar{m}_2+w)^{\delta_1} + \alpha_2\beta_2(\beta_2+1)(w-c)(\bar{m}_2+w)^{-\beta_2-2}(\bar{m}_1+w)^{\delta_2} \\
- &\quad 2\alpha_1\beta_1\delta_1(w-c)(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1-1} - 2\alpha_2\beta_2\delta_2(w-c)(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2-1} \\
- &\quad 2\alpha_1\beta_1(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1} + 2\alpha_1\delta_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-1} \\
- &\quad 2\alpha_2\beta_2(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2} + 2\alpha_2\delta_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-1}]\,| \\
+ & \\
&\quad |\,[\alpha_1\delta_1(\delta_1-1)(w-c)(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-2} - \alpha_1\beta_1\delta_1(w-c)(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1-1} \\
+ &\quad \alpha_1\delta_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-1} - \alpha_2\beta_2\delta_2(w-c)(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2-1} \\
+ &\quad \alpha_2\beta_2(\beta_2+1)(w-c)(\bar{m}_2+w)^{-\beta_2-2}(\bar{m}_1+w)^{\delta_2} - \alpha_2\beta_2(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2}] \\
/ &\quad [\alpha_1\delta_1(\delta_1-1)(w-c)(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-2} + \alpha_2\delta_2(\delta_2-1)(w-c)(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-2} \\
+ &\quad \alpha_1\beta_1(\beta_1+1)(w-c)(\bar{m}_1+w)^{-\beta_1-2}(\bar{m}_2+w)^{\delta_1} + \alpha_2\beta_2(\beta_2+1)(w-c)(\bar{m}_2+w)^{-\beta_2-2}(\bar{m}_1+w)^{\delta_2} \\
- &\quad 2\alpha_1\beta_1\delta_1(w-c)(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1-1} - 2\alpha_2\beta_2\delta_2(w-c)(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2-1} \\
- &\quad 2\alpha_1\beta_1(\bar{m}_1+w)^{-\beta_1-1}(\bar{m}_2+w)^{\delta_1} + 2\alpha_1\delta_1(\bar{m}_1+w)^{-\beta_1}(\bar{m}_2+w)^{\delta_1-1} \\
- &\quad 2\alpha_2\beta_2(\bar{m}_2+w)^{-\beta_2-1}(\bar{m}_1+w)^{\delta_2} + 2\alpha_2\delta_2(\bar{m}_2+w)^{-\beta_2}(\bar{m}_1+w)^{\delta_2-1}]\,| < 1. \qquad (4.17)
\end{aligned}
$$

When (4.14) and (4.17) hold, there is a unique equilibrium for the nonlinear demand model. It was shown that (4.14) will always be true. However, due to the difficulty in simplifying (4.17), numerical procedures were carried out to determine uniqueness of Cournot equilibrium. Each example used for the sensitivity analysis in Chapter 5 and shown in Appendix D is unique.

CHAPTER 5. SENSITIVITY ANALYSIS OF NONLINEAR DEMAND

This chapter summarizes the results of the constant elasticity nonlinear demand model. Sensitivity analysis was performed to determine the effects that four parameters had on equilibrium margins, prices, quantities, and profits. The parameters, which include market potential, price elasticity of demand, manufacturing variable cost, and cross price elasticity of demand, lead the discussion for the analysis of the nonlinear demand model. Observations pertaining to the sensitivity analysis will be conveyed, in addition to the economic and managerial implications for the manufacturer, retailers, and consumers.

The fundamental parameter values are $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, and $\delta_2 = .20$. β_1 was varied from 3, which is near its lower limit, to 8, a value that led to small profits. δ_1 was varied between its upper and lower limits, $1 > \delta_1 > 0$. Additional parameters were allowed to differ by $\pm 10\%$, $\pm 20\%$, and $\pm 50\%$, as seen in Arcelus & Srinivasan (1989). Using these parameter values and equations (4.2), (4.3), (4.4), (4.5) and (4.7), the equilibrium quantities, profits, margins, and prices were found.

The examples used for the sensitivity analysis are shown in Appendix D. Each example used in this analysis satisfies the sufficient conditions for existence and uniqueness of Cournot equilibrium as conveyed in Chapter 4. Equilibrium decisions for the two-echelon model, total channel profit, and RT, the ratio of the manufacturer's profit to the total profit of the retailers, will be discussed in the following sections.

5.1 Absolute Market Potential

The parameter for absolute market potential was varied while all other parameters were held constant. The equilibrium solutions for margin, price, quantity, and profit are shown in

Figures 5.1, 5.2, 5.3, and 5.4 and lead the analysis. The conclusions made for retailer 1 would be identical to the conclusions made for retailer 2, had absolute market potential for retailer 2 been altered and all other parameters held constant.

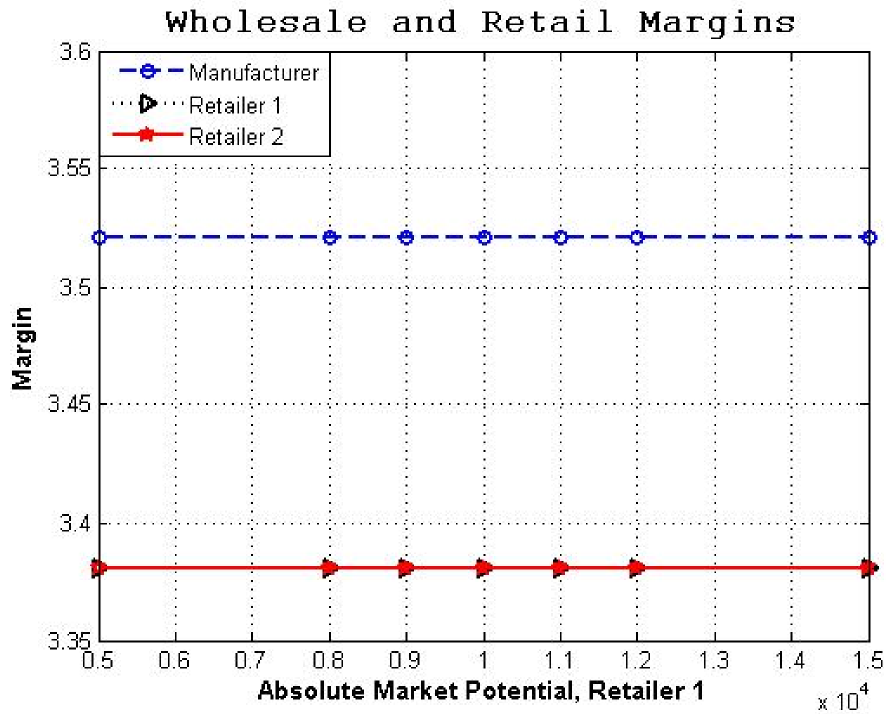

Figure 5.1 α_1: $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

Figures 5.1 and 5.2 show that retail margins and retail prices are identical and independent of market potential; refer to equation (4.7). For these reasons, consumers do not have a preference as to the size of the market.

In Figure 5.3, there is a direct correlation between market potential and quantity demanded. Retailers will be rewarded with large demand if they start out with a wide market. Furthermore, the retailer with larger market potential has a larger demand. Figure 5.3 also shows that the market potential for retailer 1 has no influence over the quantity demanded for retailer 2.

Figure 5.4 shows that profits for the manufacturer and retailer 1 increase with market potential. Using the cheese example from Chapter 1, when retailer 1 has a wide market for cheese, retailer 1 will have large profits due to large customer demand. Additionally, the manufacturer will earn large profits by producing a large quantity of cheese for retailer 1.

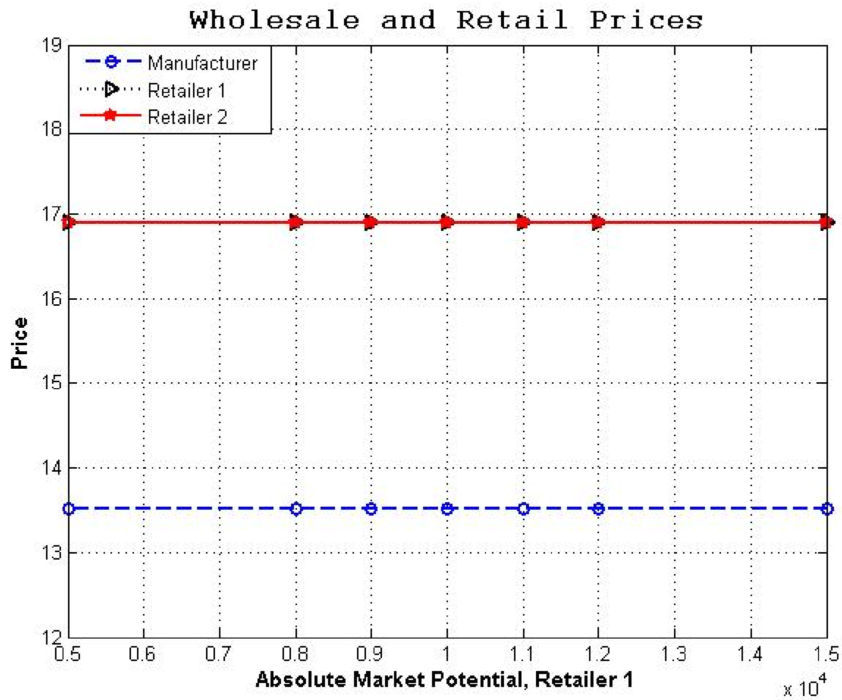

Figure 5.2 α_1: $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

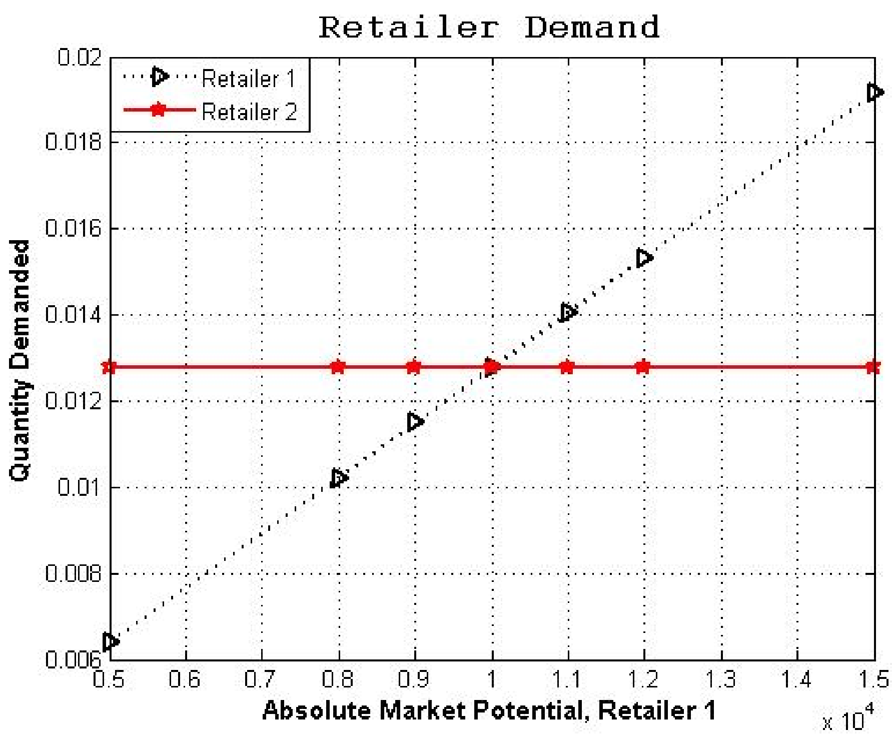

Figure 5.3 α_1: $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

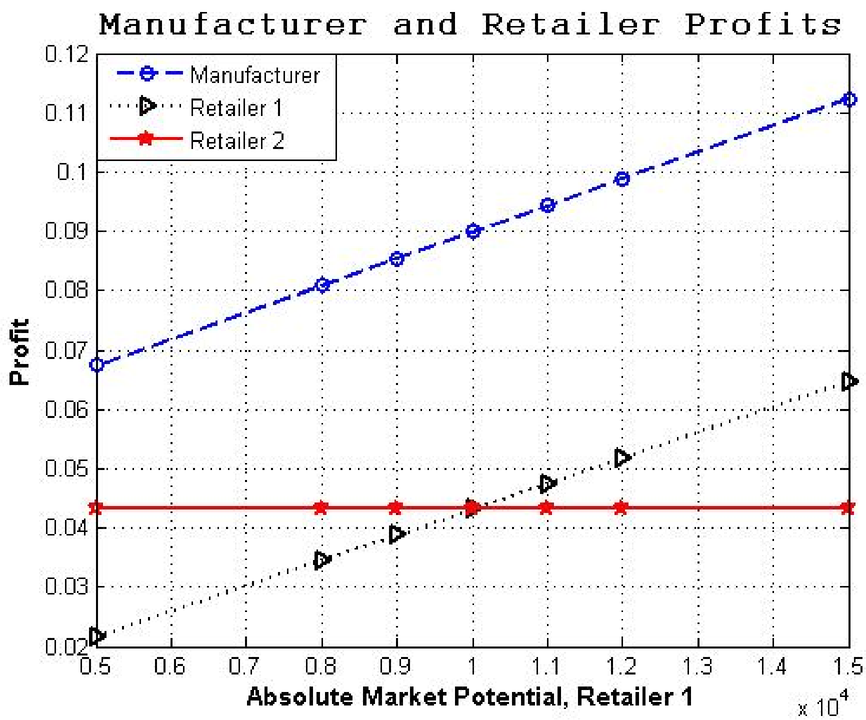

Figure 5.4 α_1: $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

Figure 5.4 also shows that the retailer with larger market potential has a larger profit. Since margin and quantity demanded were constant for retailer 2, profit is as well. Appendix D shows that RT stays constant regardless of market potential. Additionally, total profit increases as market potential increases.

The observations relating to quantity and profit follow the intuition regarding market potential. It is expected that these values increase as market potential increases. However, a constant price for all levels of market size does bring about a possible weakness with the constant elasticity nonlinear demand model. Firms would most likely raise prices if market potential increased. This way, more profit could be made. With the constant elasticity nonlinear demand model, as market potential increases, the manufacturer and retailer 1 benefit from increasing profits due to increasing demand and not increasing prices.

5.2 Price Elasticity of Demand

Price elasticity of demand is a measure of sensitivity or responsiveness relating quantity demanded to price. Because of the constraint $\beta_i > 2$, price elasticity of demand will always be elastic. As β_i increases, quantity demanded for retailer i becomes more and more sensitive to changes in retailer i's price. The conclusions made for retailer 1 would be identical to those made for retailer 2, had price elasticity of demand for retailer 2 been studied.

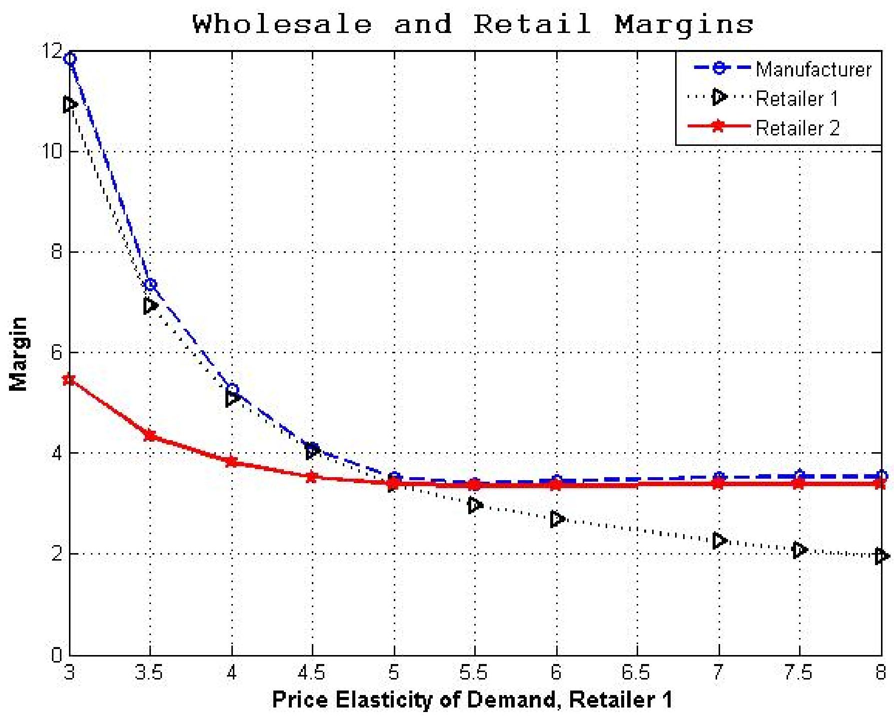

Figure 5.5 β_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

Figures 5.5 and 5.6 show the margins and prices for the supply chain members. Retailer 1 has a decreasing margin and price as its elasticity increases. The manufacturer and retailer 2 have either a decreasing or increasing margin and price as price elasticity for retailer 1 increases. The minimum margin and price for the manufacturer and retailer 2 occur at the same level of price elasticity for retailer 1. The maximum margin and price for all players occur when price elasticity for retailer 1 is lowest. The retailer with the lower value of price elasticity has the higher margin and higher price.

Figure 5.6 β_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

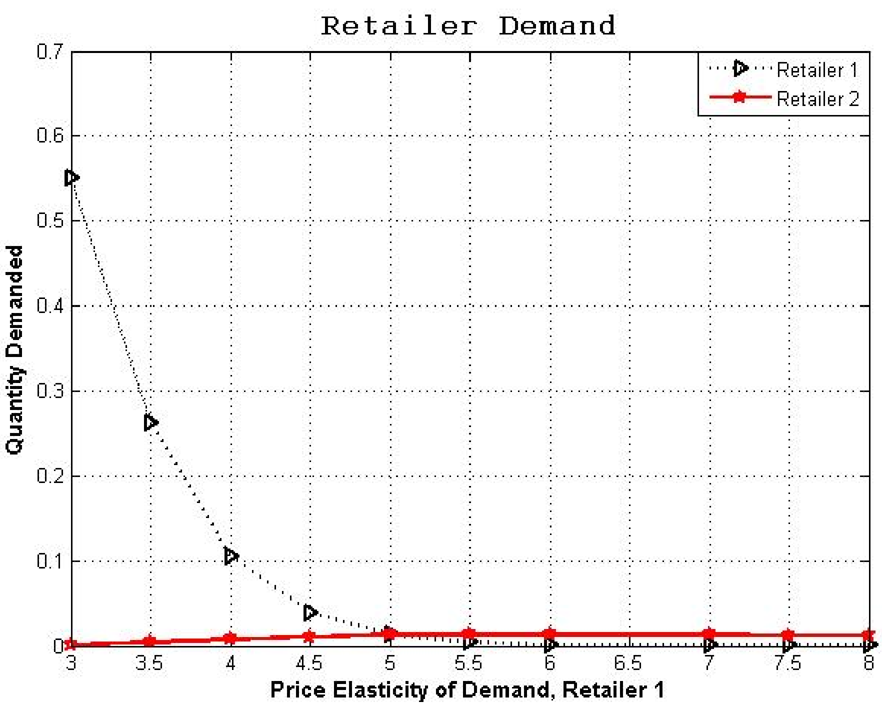

Figure 5.7 β_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

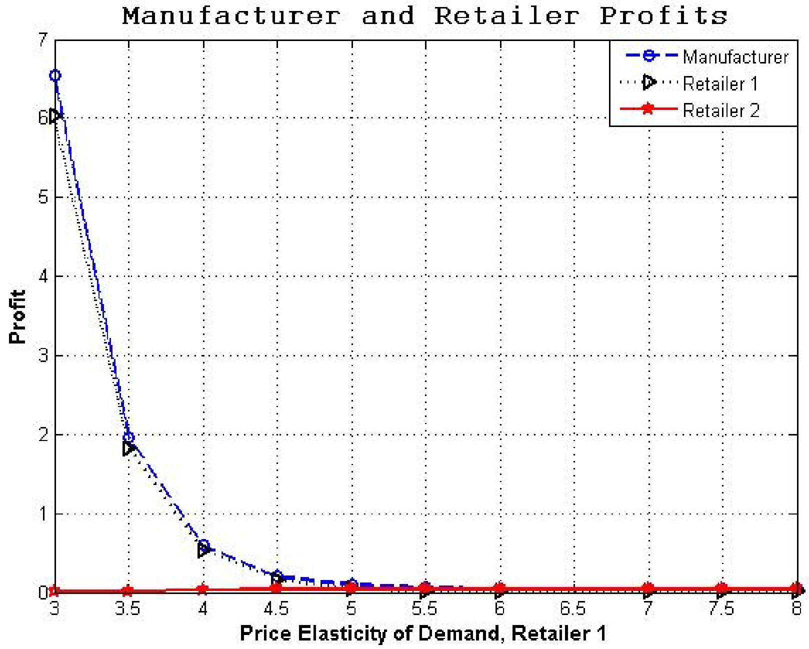

Figure 5.8 β_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_2 = 5.0$, $c = 10$, $\delta_1 = .20$, $\delta_2 = .20$

The maximum quantity demanded for each retailer occurs at different prices and at different levels of price elasticity. Demand for retailer 1 is large when price is high and price elasticity of demand is low. Using the cheese example from Chapter 1, if cheese has a low price elasticity of demand, there will still be a large demand for it even though prices are high. Demand then decreases with increasing price elasticity. Retailer 2 has its largest demand at its lowest price.

Since price elasticity of demand is elastic, quantity demanded will respond to changes in price. This is exhibited by retailer 2 in Figures 5.6 and 5.7 and in Appendix D. When price significantly decreases, quantity demanded significantly increases. When there is a small increase in price, there is a small decrease in demand.

Figure 5.8 and Appendix D show that the retailer with lower price elasticity of demand has a larger profit. Retailer 1 has its largest profit when price and quantity demanded are highest, which occurs when its price elasticity is lowest. Retailer 2 has its largest profit when price is at its lowest and quantity demanded is at its highest. This occurs when price elasticity of demand for the retailers is nearly symmetric. RT decreases and then increases for increasing levels of price elasticity.

The profits for the manufacturer, retailer 1, and the total channel are largest when price elasticity of demand for retailer 1 is lowest. *RT* is also highest when price elasticity of demand for retailer 1 is lowest. These observations can be obtained using Appendix D. Retailer 2 prefers price elasticity of demand for retailer 1 to be slightly above its own price elasticity. Retailer 2 has its largest profit when this occurs. Based on retail price, consumers will want to buy the substitute product that has the higher price elasticity of demand. In addition, as price elasticity of demand increases, consumer surplus increases.

5.3 Manufacturing Variable Cost

Manufacturing variable cost was varied to determine the effects it has on the equilibrium solutions for margin, price, quantity, and profit. Figures 5.9, 5.10, 5.11, and 5.12 display these solutions.

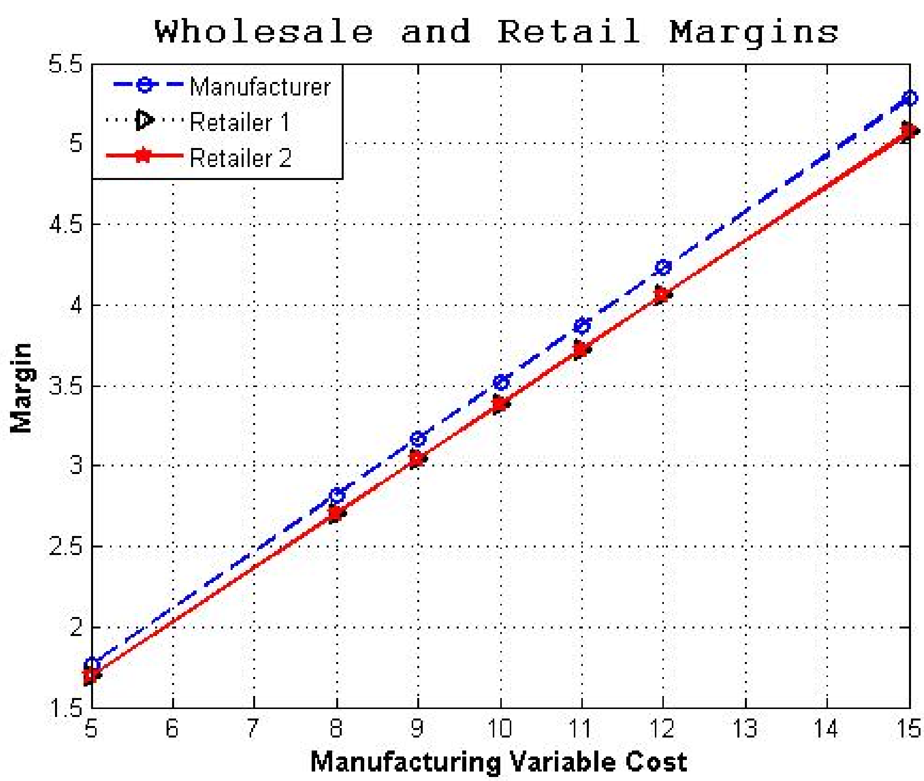

Figure 5.9 c: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $\delta_1 = .20$, $\delta_2 = .20$

Figure 5.10 c: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $\delta_1 = .20$, $\delta_2 = .20$

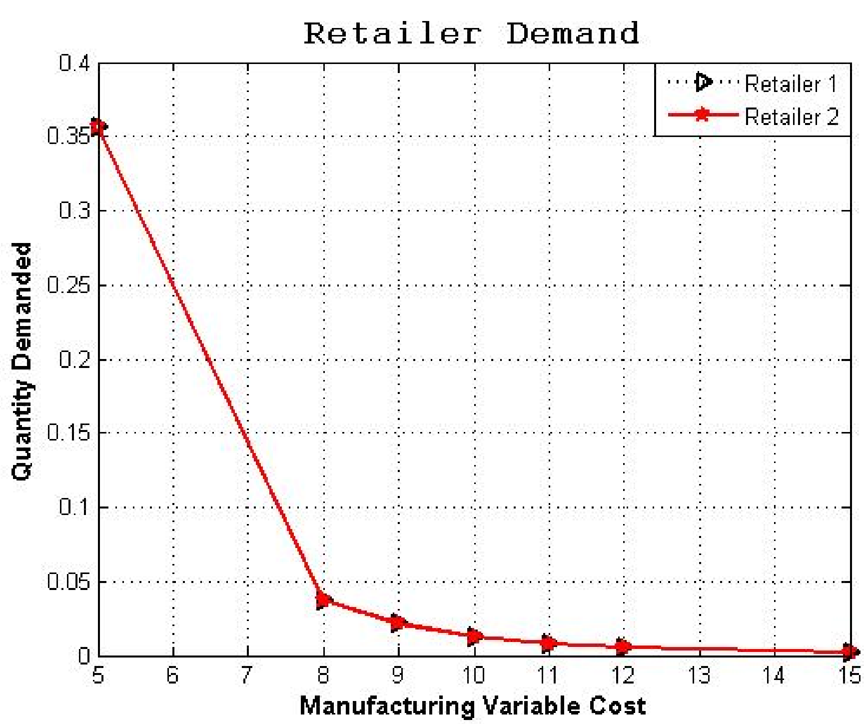

Figure 5.11 c: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $\delta_1 = .20$, $\delta_2 = .20$

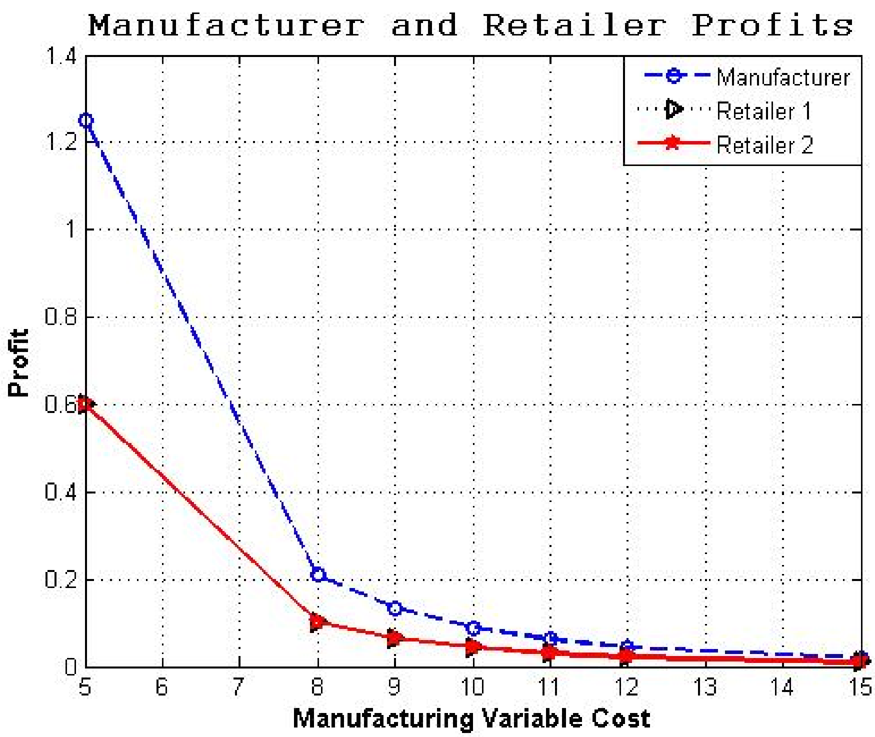

Figure 5.12 c: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $\delta_1 = .20$, $\delta_2 = .20$

As manufacturing variable cost increases, all margins and prices increase. With the increasing manufacturing variable cost comes decreasing demand, as shown in Figure 5.11. The decreasing demand has a greater effect than the increasing retail margins, resulting in decreasing retail profits. The manufacturer also has decreasing profits when manufacturing variable cost increases.

Retailers have identical margins, prices, quantity demanded, and profits regardless of the manufacturing variable cost. This is confirmed by the equations in (4.8), (4.9), and (4.10). Also, RT remains constant regardless of manufacturing variable cost. This is derived from equation (4.10) and shown in Appendix D.

The entire oligopoly benefits from low manufacturing variable cost. Even though margins and prices are low when manufacturing variable cost is low, profits are still high because demand is large. Consumers benefit from low retail prices when manufacturing variable cost is low.

The results from this section reflect the impact of cost reduction. When a firm cuts costs, it can gain a competitive advantage, which leads to large demand and substantial profit. For example, if a manufacturer is able to reduce its manufacturing variable cost to produce cheese, the manufacturer will be able to offer the cheese to retailers at a lower wholesale price. In turn, retailers will be able to sell the cheese to customers at a lower retail price. The lower retail prices lead to increasing customer demand. The increasing customer demand creates large profits for the retailers and a large demand and large profit for the manufacturer.

5.4 Cross Price Elasticity of Demand

Cross price elasticity of demand is similar to price elasticity of demand in that it measures the sensitivity of quantity demanded to price. However, cross price elasticity measures changes across retailers. With substitutable products, increases/decreases in retailer i's price will cause demand for retailer j to increase/decrease. The conclusions for retailer 2 would be identical to those made for retailer 1, had cross price elasticity of demand for retailer 2 been explored.

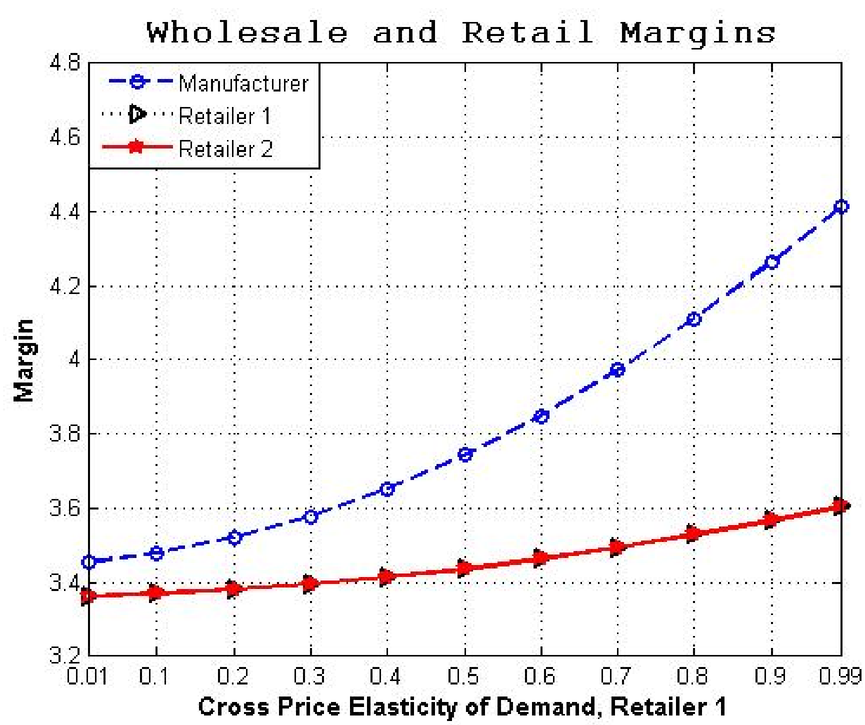

Figure 5.13 δ_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_2 = .20$

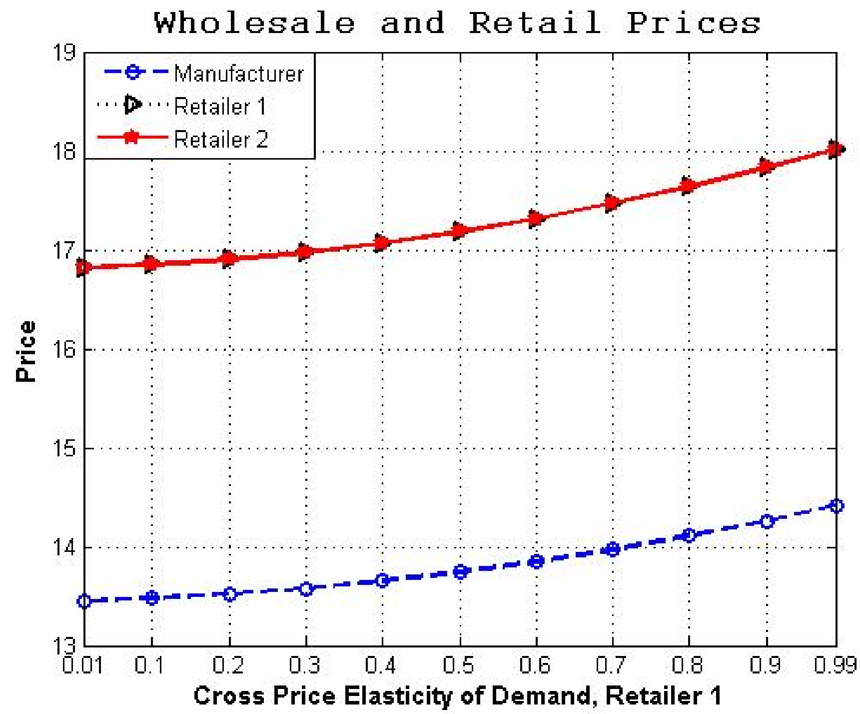

Figure 5.14 δ_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_2 = .20$

Figures 5.13 and 5.14 illustrate that margins and prices increase as cross price elasticity of demand for retailer 1 increases. Retail margins and retail prices are identical regardless of cross price elasticity of demand; refer to equation (4.7). Demand for retailer 1 increases and demand for retailer 2 decreases as cross price elasticity of demand for retailer 1 increases. This is demonstrated in Figure 5.15 and Appendix D.

A similar relationship can be seen in Figure 5.16. Profit for retailer 1 increases and profit for retailer 2 decreases as cross price elasticity of demand for retailer 1 increases. As for the manufacturer, profit increases with cross price elasticity of demand. Total channel profit is large when cross price elasticity of demand for retailer 1 is high. Conversely, consumers prefer products with a low cross price elasticity of demand because of the low retail prices.

The following example illustrates the effects of cross price elasticity of demand. Suppose retailer 1 sells Brand A cheese and retailer 2 sells Brand B cheese. If Brand A cheese has a high cross price elasticity of demand, increases in price for Brand B cheese will lead to increases

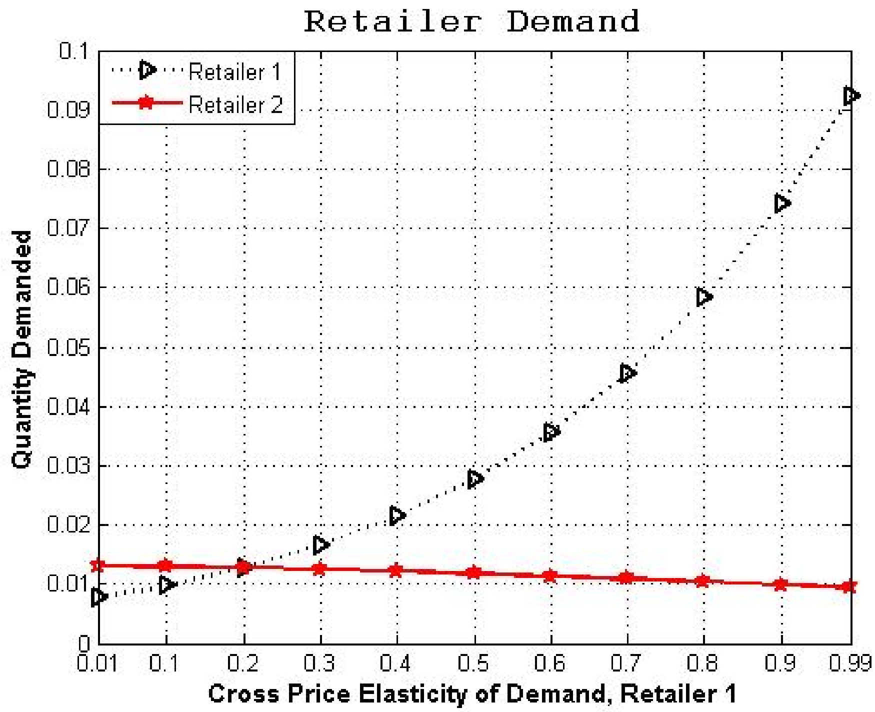

Figure 5.15 δ_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_2 = .20$

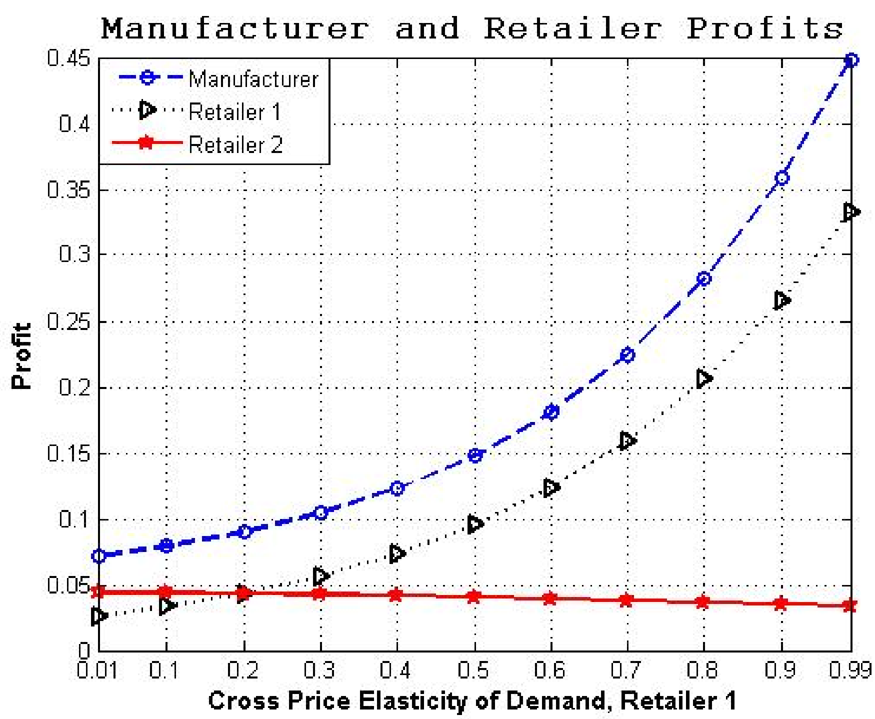

Figure 5.16 δ_1: $\alpha_1 = 10000$, $\alpha_2 = 10000$, $\beta_1 = 5.0$, $\beta_2 = 5.0$, $c = 10$, $\delta_2 = .20$

in demand for Brand A cheese. Retailer 1 will then request that the manufacturer produce a larger quantity of Brand A cheese. Because demand increases for the manufacturer and for retailer 1, profits increase for the both of them as well. As for retailer 2, when the price for Brand B cheese increases, demand for Brand B cheese decreases. Consequently, profit for retailer 2 decreases.

CHAPTER 6. CONCLUSION

This paper extends the current literature by examining the margin, price, and quantity decisions made by one manufacturer and two retailers under Cournot competition. The sufficient conditions for existence and uniqueness of Cournot equilibrium for linear and nonlinear demand functions are shown. In addition, the effects of initial demand, price effect, manufacturing variable cost, and cross price effect for a linear demand model and market potential, price elasticity of demand, manufacturing variable cost, and cross price elasticity of demand for a nonlinear demand model are analyzed. Subsequent paragraphs summarize important findings of this paper and outline future research opportunities.

Some parameters, such as price effect in the linear demand model, presented important findings. The retailer with the lower price effect has a higher margin, price, quantity demanded, and profit. Similarly, for the nonlinear demand model, it was found that the retailer with the lower price elasticity of demand has a higher margin, price, quantity demanded, and profit. So, retailers will benefit by selling products that have a low sensitivity to price.

The theoretical results of this paper pertaining to price sensitivity help a manufacturer determine which product to produce and help retailers determine which product to sell. For instance, suppose a manufacturer produces a brand of cheese with a high price effect or high price elasticity of demand. As shown in this paper, the manufacturer would benefit by producing a different brand of cheese or a different product altogether if it had a lower price effect or lower price elasticity of demand. Retailers, such as grocery stores, also benefit from the results of this paper; retailers should sell a product that has a low price effect or low price elasticity of demand.

Cross price effect in the linear demand model and cross price elasticity of demand in the nonlinear demand model are two additional parameters with important implications. Profits are large for all members of the supply chain when cross price effect is high. In addition, the retailer that sells the product with the higher cross price elasticity of demand will have larger demand and larger profit. So, retailers will benefit by selling products that have a high sensitivity to a competitor's price.

Retailers can determine which products to sell using the theoretical results of this paper regarding cross price sensitivity. For example, suppose a grocery store sells a brand of cheese with a low cross price effect or low cross price elasticity of demand. The grocery store would benefit if it sold a different brand of cheese or different product that had a higher cross price effect or higher cross price elasticity of demand. In addition, the manufacturer should produce a product with a high cross price effect or high cross price elasticity of demand.

Two parameters uncovered weaknesses with their respective demand functions. With linear demand, it was found that equilibrium prices increase for the manufacturer and retailers as products become more substitutable. This result contradicts intuition and may be due, as Choi (1991) describes it, to the symmetric linear demand functions. As for nonlinear demand, all equilibrium prices are constant as absolute market potential changes. This is a counterintuitive result because price is expected to respond to changes in the size of the market.

Even though this paper enhances the current literature on game theory and the supply chain, further research is warranted. One of the main assumptions of this paper was that the manufacturer sells substitutable products to two retailers for the same wholesale price. An additional model that could be studied is one in which the manufacturer sells substitutable products to two retailers for a different wholesale price. This additional model would add to the number of applications of the current model.

Furthermore, empirical studies could be conducted using the models of this paper to determine the types of products that have low price sensitivity and high cross price sensitivity. This will help the manufacturer determine the products to produce and help the retailers determine the products to sell.

Finally, this paper could be extended to include a general demand function. This paper, along with previous literature such as Lau & Lau (2003), demonstrate the importance of utilizing the correct demand function; different results are achieved with different demand functions. Thus, it is imperative that players understand the demand function faced by their firm or industry. A model that included a general demand function would provide a benchmark for firms in Cournot competition.

APPENDIX A. Example: Existence of Cournot Equilibrium

The following example illustrates the three sufficient conditions for the existence of Cournot equilibrium. The two players in this example include one profit maximizing manufacturer and one profit maximizing retailer. The objective functions for the manufacturer and retailer are

$$\begin{aligned} \max_{w} \Pi_M &= 16w + 12w^3 - 4w^4 - 4w\bar{m} + 12\bar{m} - 48, \\ \max_{m} \Pi_R &= 11m - \frac{m^2}{5} - 4m\bar{w}. \end{aligned} \tag{A.1}$$

The two reaction functions then become

$$\begin{aligned} w(\bar{m}) &= \frac{(59 - 8\bar{m} + 4\sqrt{172 - 59\bar{m} + 4\bar{m}^2})^{2/3} + 9 + 3(59 - 8\bar{m} + 4\sqrt{172 - 59\bar{m} + 4\bar{m}^2})^{1/3}}{4(59 - 8\bar{m} + 4\sqrt{172 - 59\bar{m} + 4\bar{m}^2})^{1/3}}, \\ m(\bar{w}) &= 27.5 - 10\bar{w}, \end{aligned} \tag{A.2}$$

and are shown in Figure A.1. Since the reaction functions intersect, the first condition for the existence of Cournot equilibrium is satisfied. It turns out that the reaction functions intersect twice. The next two conditions will determine if there are multiple equilibria for this game.

The second condition is that the assumptions each firm makes about another firm's actions must be confirmed as the actual behavior. Figure A.1 displays the reaction functions for the manufacturer and retailer when they both have correct assumptions regarding each other's price or margin.

Finally, the second order sufficient condition for the manufacturer and retailer is

$$\begin{aligned} \frac{\partial^2 \Pi_M}{\partial w^2} &= -24w(2w - 3) < 0, \\ \frac{\partial^2 \Pi_R}{\partial m^2} &= -\frac{2}{5} < 0. \end{aligned} \tag{A.3}$$

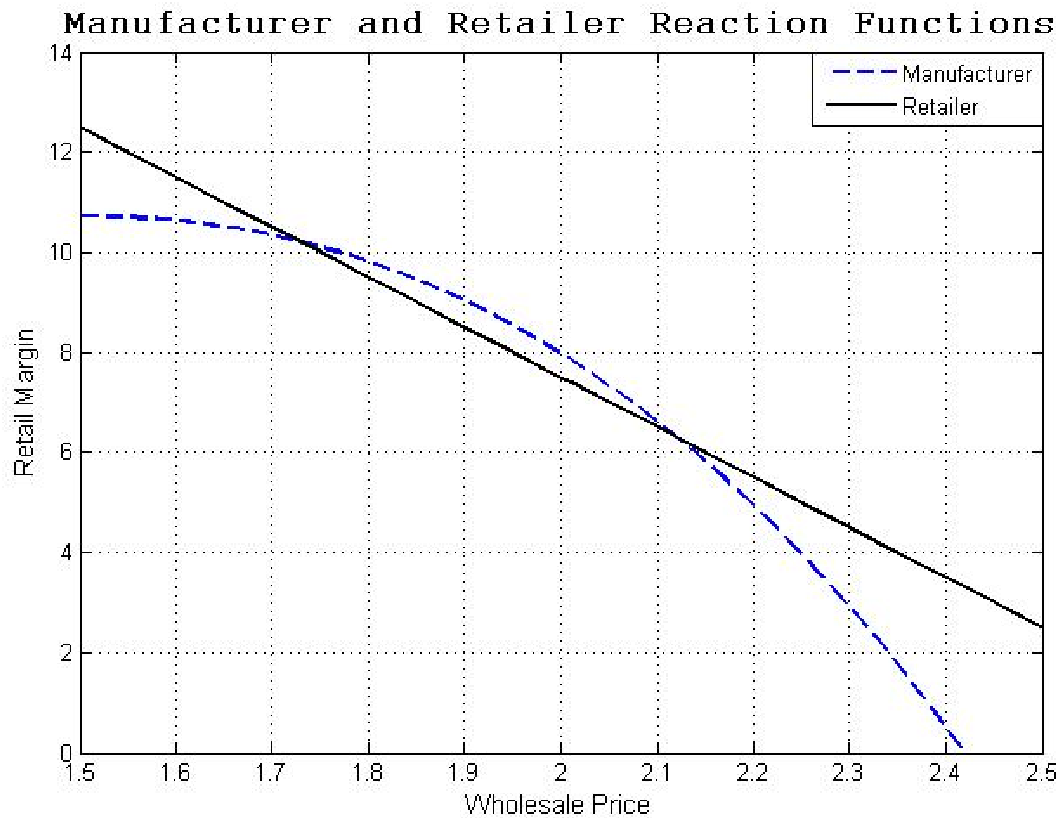

Figure A.1 Multiple Equilibria

The manufacturer maximizes profit and has a concave profit function when $w > 1.5$. The retailer has a concave profit function regardless of the retail margin. So, the three conditions for the existence of Cournot equilibrium are satisfied for two different equilibria.

APPENDIX B. Example: Uniqueness of Cournot Equilibrium

This section is the continuation of the example from Appendix A that illustrated a game with multiple equilibria. Since the game has multiple equilibria, the condition for the uniqueness of Cournot equilibrium fails and is demonstrated below.

In order to verify that a game has a unique Cournot equilibrium, it is sufficient to show that the reaction function for each player is a contraction. Using the retailer's reaction function, (A.2) in Appendix A,

$$\left|\frac{\partial m(\bar{w})}{\partial \bar{w}}\right| = \left|-10\right|. \tag{B.1}$$

Since $|-10| \geq 1$, the retailer's reaction function is not a contraction. The same conclusion can be made from Figure B.1. When the contraction mapping approach is applied to the retailer's reaction function, there is diverging iterations. The game does not have a unique equilibrium, and this confirms the result in Appendix A.

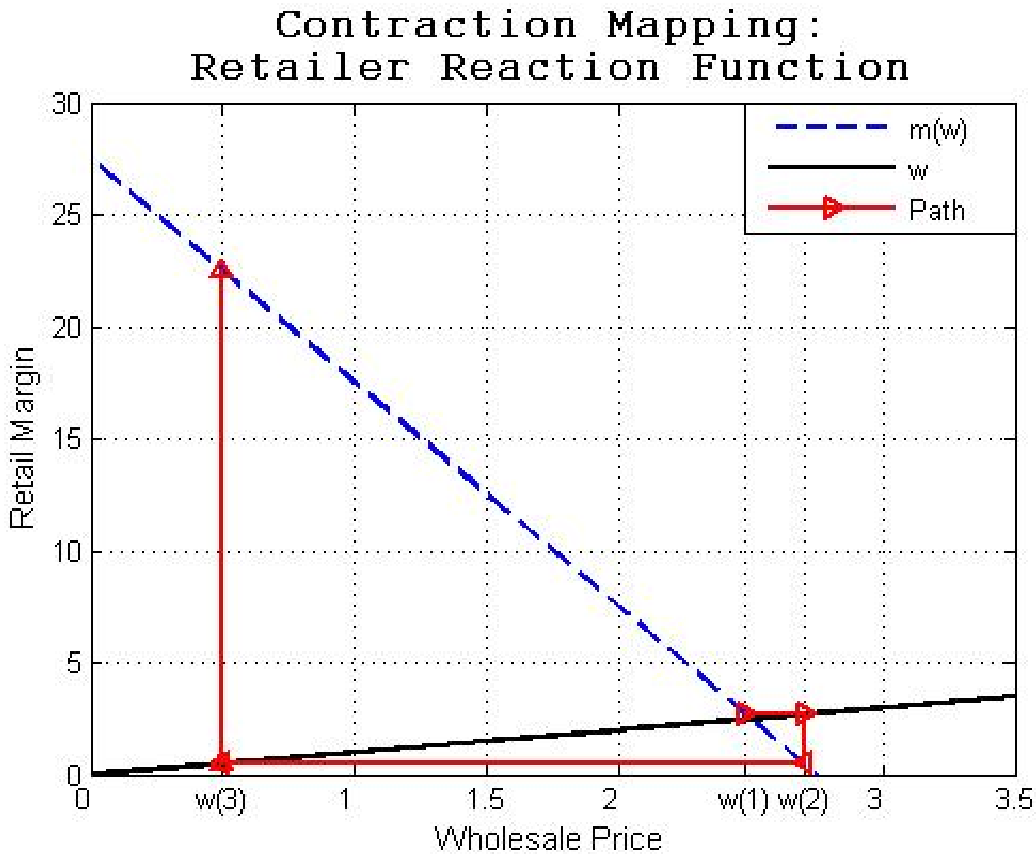

Figure B.1 Diverging Iterations

APPENDIX C. Examples: Linear Model

Ex	a_1	a_2	b_1	b_2	c	γ	$\frac{\gamma^2}{b_1 b_2}$	$\hat{m}^*$	${m_1}^*$	${m_2}^*$	w^*	${p_1}^*$	${p_2}^*$	${q_1}^*$	${q_2}^*$	Π_M^*	$\Pi_{R_1}^*$	$\Pi_{R_2}^*$	RT	Π_C^*
1	50	100	5.0	5.0	10	2.0	0.1600	5.77	1.38	5.54	15.77	17.15	21.31	6.89	27.72	199.70	9.50	153.73	1.223	362.93
2	80	100	5.0	5.0	10	2.0	0.1600	7.69	3.78	5.45	17.69	21.47	23.14	18.91	27.24	355.03	71.52	148.44	1.614	574.99
3	90	100	5.0	5.0	10	2.0	0.1600	8.33	4.58	5.42	18.33	22.92	23.75	22.92	27.08	416.67	105.03	146.70	1.655	668.40
4	100	100	5.0	5.0	10	2.0	0.1600	8.97	5.38	5.38	18.97	24.36	24.36	26.92	26.92	483.23	144.97	144.97	1.667	773.18
5	110	100	5.0	5.0	10	2.0	0.1600	9.62	6.19	5.35	19.62	25.80	24.97	30.93	26.76	554.73	191.33	143.25	1.658	889.31
6	120	100	5.0	5.0	10	2.0	0.1600	10.26	6.99	5.32	20.26	27.24	25.58	34.94	26.60	631.16	244.10	141.54	1.637	1016.81
7	150	100	5.0	5.0	10	2.0	0.1600	12.18	9.39	5.22	22.18	31.57	27.40	46.96	26.12	890.04	440.96	136.47	1.541	1467.47
8	100	100	2.5	5.0	10	2.0	0.3200	20.20	18.86	4.71	30.20	49.07	34.92	47.16	23.56	1428.82	889.63	110.98	1.428	2429.44
9	100	100	4.0	5.0	10	2.0	0.2000	11.79	8.33	5.13	21.79	30.13	26.92	33.33	25.64	695.60	277.78	131.49	1.700	1104.87
10	100	100	4.5	5.0	10	2.0	0.1778	10.23	6.66	5.26	20.23	26.89	25.50	29.97	26.31	575.93	199.61	138.45	1.704	913.99
11	100	100	5.0	5.0	10	2.0	0.1600	8.97	5.38	5.38	18.97	24.36	24.36	26.92	26.92	483.23	144.97	144.97	1.667	773.18
12	100	100	5.5	5.0	10	2.0	0.1455	7.94	4.38	5.50	17.94	22.32	23.43	24.11	27.48	409.40	105.67	151.02	1.595	666.08
13	100	100	6.0	5.0	10	2.0	0.1333	7.06	3.58	5.60	17.06	20.64	22.66	21.47	27.98	349.32	76.81	156.59	1.497	582.73
14	100	100	7.5	5.0	10	2.0	0.1067	5.12	1.90	5.85	15.12	17.02	20.96	14.27	29.23	222.61	27.16	170.84	1.124	420.61
15	100	100	5.0	5.0	1	2.0	0.1600	12.44	7.46	7.46	13.44	20.90	20.90	37.31	37.31	927.91	278.37	278.37	1.667	1484.65
16	100	100	5.0	5.0	4	2.0	0.1600	11.28	6.77	6.77	15.28	22.05	22.05	33.85	33.85	763.71	229.11	229.11	1.667	1221.93
17	100	100	5.0	5.0	7	2.0	0.1600	10.13	6.08	6.08	17.13	23.21	23.21	30.38	30.38	615.48	184.64	184.64	1.667	984.77
18	100	100	5.0	5.0	10	2.0	0.1600	8.97	5.38	5.38	18.97	24.36	24.36	26.92	26.92	483.23	144.97	144.97	1.667	773.18
19	100	100	5.0	5.0	13	2.0	0.1600	7.82	4.69	4.69	20.82	25.51	25.51	23.46	23.46	366.96	110.09	110.09	1.667	587.14
20	100	100	5.0	5.0	16	2.0	0.1600	6.67	4.00	4.00	22.67	26.67	26.67	20.00	20.00	266.67	80.00	80.00	1.667	426.67
21	100	100	5.0	5.0	19	2.0	0.1600	5.51	3.31	3.31	24.51	27.82	27.82	16.54	16.54	182.35	54.70	54.70	1.667	291.76
22	100	100	5.0	5.0	22	2.0	0.1600	4.36	2.62	2.62	26.36	28.97	28.97	13.08	13.08	114.00	34.20	34.20	1.667	182.41
23	100	100	5.0	5.0	25	2.0	0.1600	3.21	1.92	1.92	28.21	30.13	30.13	9.62	9.62	61.64	18.49	18.49	1.667	98.62
24	100	100	5.0	5.0	28	2.0	0.1600	2.05	1.23	1.23	30.05	31.28	31.28	6.15	6.15	25.25	7.57	7.57	1.667	40.39
25	100	100	5.0	5.0	31	2.0	0.1600	0.90	0.54	0.54	31.90	32.44	32.44	2.69	2.69	4.83	1.45	1.45	1.667	7.73
26	100	100	5.0	5.0	10	0.1	0.0004	3.49	3.42	3.42	13.49	16.92	16.92	17.11	17.11	119.55	58.58	58.58	1.020	236.70
27	100	100	5.0	5.0	10	0.5	0.0100	4.21	3.79	3.79	14.21	18.01	18.01	18.97	18.97	159.86	71.94	71.94	1.111	303.74
28	100	100	5.0	5.0	10	0.9	0.0324	5.10	4.18	4.18	15.10	19.29	19.29	20.92	20.92	213.53	87.55	87.55	1.220	388.62
29	100	100	5.0	5.0	10	1.3	0.0676	6.21	4.60	4.60	16.21	20.81	20.81	22.99	22.99	285.76	105.73	105.73	1.351	497.23
30	100	100	5.0	5.0	10	1.7	0.1156	7.63	5.04	5.04	17.63	22.67	22.67	25.19	25.19	384.51	126.89	126.89	1.515	638.28
31	100	100	5.0	5.0	10	2.1	0.1764	9.49	5.50	5.50	19.49	24.99	24.99	27.52	27.52	522.29	151.46	151.46	1.724	825.21
32	100	100	5.0	5.0	10	2.5	0.2500	12.00	6.00	6.00	22.00	28.00	28.00	30.00	30.00	720.00	180.00	180.00	2.000	1080.00
33	100	100	5.0	5.0	10	2.9	0.3364	15.55	6.53	6.53	25.55	32.07	32.07	32.64	32.64	1014.93	213.13	213.13	2.381	1441.19
34	100	100	5.0	5.0	10	3.3	0.4356	20.86	7.09	7.09	30.86	37.96	37.96	35.47	35.47	1480.15	251.63	251.63	2.941	1983.40
35	100	100	5.0	5.0	10	3.7	0.5476	29.61	7.70	7.70	39.61	47.31	47.31	38.50	38.50	2279.86	296.38	296.38	3.846	2872.62
36	100	100	5.0	5.0	10	4.1	0.6724	46.38	8.35	8.35	56.38	64.73	64.73	41.74	41.74	3872.20	348.50	348.50	5.556	4569.19
37	100	100	5.0	5.0	10	4.5	0.8100	90.48	9.05	9.05	100.48	109.52	109.52	45.24	45.24	8185.94	409.30	409.30	10.000	9004.54

APPENDIX D. Examples: Nonlear Model

Ex	α_1	α_2	β_1	β_2	c	δ_1	δ_2	$\hat{m}^*$	${m_1}^*$	${m_2}^*$	w^*	${p_1}^*$	${p_2}^*$	${q_1}^*$	${q_2}^*$	Π_M^*	$\Pi_{R_1}^*$	$\Pi_{R_2}^*$	RT	Π_C^*
1	5000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.006382	0.012764	0.06741	0.02157	0.04314	1.042	0.13
2	8000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.010211	0.012764	0.08090	0.03452	0.04314	1.042	0.16
3	9000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.011487	0.012764	0.08539	0.03883	0.04314	1.042	0.17
4	10000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.012764	0.012764	0.08988	0.04314	0.04314	1.042	0.18
5	11000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.014040	0.012764	0.09438	0.04746	0.04314	1.042	0.18
6	12000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.015316	0.012764	0.09887	0.05177	0.04314	1.042	0.19
7	15000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.019145	0.012764	0.11235	0.06472	0.04314	1.042	0.22
8	10000	10000	3.0	5.0	10	0.20	0.20	11.84	10.92	5.46	21.84	32.75	27.30	0.551329	0.001326	6.54135	6.01947	0.00724	1.085	12.57
9	10000	10000	3.5	5.0	10	0.20	0.20	7.34	6.94	4.33	17.34	24.28	21.67	0.262500	0.003957	1.95561	1.82063	0.01715	1.064	3.79
10	10000	10000	4.0	5.0	10	0.20	0.20	5.25	5.08	3.81	15.25	20.33	19.06	0.105506	0.007258	0.59193	0.53630	0.02767	1.050	1.16
11	10000	10000	4.5	5.0	10	0.20	0.20	4.09	4.02	3.52	14.09	18.11	17.61	0.038775	0.010549	0.20150	0.15605	0.03715	1.043	0.39
12	10000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.012764	0.012764	0.08988	0.04314	0.04314	1.042	0.18
13	10000	10000	5.5	5.0	10	0.20	0.20	3.39	2.98	3.35	13.39	16.37	16.74	0.003693	0.013293	0.05766	0.01099	0.04451	1.039	0.11
14	10000	10000	6.0	5.0	10	0.20	0.20	3.44	2.69	3.36	13.44	16.13	16.80	0.000998	0.013020	0.04825	0.00268	0.04375	1.039	0.09
15	10000	10000	7.0	5.0	10	0.20	0.20	3.52	2.25	3.38	13.52	15.77	16.90	0.000072	0.012589	0.04458	0.00016	0.04255	1.044	0.09
16	10000	10000	7.5	5.0	10	0.20	0.20	3.53	2.08	3.38	13.53	15.61	16.92	0.000020	0.012509	0.04426	0.00004	0.04232	1.045	0.09
17	10000	10000	8.0	5.0	10	0.20	0.20	3.54	1.93	3.38	13.54	15.47	16.92	0.000005	0.012461	0.04411	0.00001	0.04218	1.046	0.09
18	10000	10000	5.0	5.0	5	0.20	0.20	1.76	1.69	1.69	6.76	8.45	8.45	0.355561	0.355561	1.25198	0.60095	0.60095	1.042	2.45
19	10000	10000	5.0	5.0	8	0.20	0.20	2.82	2.70	2.70	10.82	13.52	13.52	0.037251	0.037251	0.20986	0.10074	0.10074	1.042	0.41
20	10000	10000	5.0	5.0	9	0.20	0.20	3.17	3.04	3.04	12.17	15.21	15.21	0.021164	0.021164	0.13414	0.06439	0.06439	1.042	0.26
21	10000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.012764	0.012764	0.08988	0.04314	0.04314	1.042	0.18
22	10000	10000	5.0	5.0	11	0.20	0.20	3.87	3.72	3.72	14.87	18.59	18.59	0.008078	0.008078	0.06257	0.03004	0.03004	1.042	0.12
23	10000	10000	5.0	5.0	12	0.20	0.20	4.23	4.06	4.06	16.23	20.28	20.28	0.005320	0.005320	0.04496	0.02158	0.02158	1.042	0.09
24	10000	10000	5.0	5.0	15	0.20	0.20	5.28	5.07	5.07	20.28	25.35	25.35	0.001823	0.001823	0.01925	0.00924	0.00924	1.042	0.04
25	10000	10000	5.0	5.0	10	0.01	0.20	3.45	3.36	3.36	13.45	16.82	16.82	0.007649	0.013077	0.07157	0.02573	0.04398	1.027	0.14
26	10000	10000	5.0	5.0	10	0.10	0.20	3.48	3.37	3.37	13.48	16.85	16.85	0.009768	0.012956	0.07906	0.03292	0.04366	1.032	0.16
27	10000	10000	5.0	5.0	10	0.20	0.20	3.52	3.38	3.38	13.52	16.90	16.90	0.012764	0.012764	0.08988	0.04314	0.04314	1.042	0.18
28	10000	10000	5.0	5.0	10	0.30	0.20	3.58	3.39	3.39	13.58	16.97	16.97	0.016600	0.012506	0.10416	0.05635	0.04245	1.054	0.20
29	10000	10000	5.0	5.0	10	0.40	0.20	3.65	3.41	3.41	13.65	17.07	17.07	0.021490	0.012185	0.12300	0.07335	0.04159	1.070	0.24
30	10000	10000	5.0	5.0	10	0.50	0.20	3.74	3.44	3.44	13.74	17.18	17.18	0.027704	0.011805	0.14788	0.09518	0.04056	1.089	0.28
31	10000	10000	5.0	5.0	10	0.60	0.20	3.85	3.46	3.46	13.85	17.31	17.31	0.035583	0.011374	0.18077	0.12320	0.03938	1.112	0.34
32	10000	10000	5.0	5.0	10	0.70	0.20	3.97	3.49	3.49	13.97	17.47	17.47	0.045561	0.010902	0.22430	0.15915	0.03808	1.137	0.42
33	10000	10000	5.0	5.0	10	0.80	0.20	4.11	3.53	3.53	14.11	17.64	17.64	0.058196	0.010400	0.28197	0.20530	0.03669	1.165	0.52
34	10000	10000	5.0	5.0	10	0.90	0.20	4.26	3.57	3.57	14.26	17.83	17.83	0.074201	0.009877	0.35842	0.26458	0.03522	1.196	0.66
35	10000	10000	5.0	5.0	10	0.99	0.20	4.41	3.60	3.60	14.41	18.02	18.02	0.092231	0.009396	0.44840	0.33232	0.03385	1.225	0.81

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